

1.  $A(1, x), B(x, 0), C(-1, 6)$  とする.

$$\vec{AC} = k\vec{BC} \text{ とする}$$

$$(-2, 6-x) = k(-1-x, 6)$$

$$\begin{cases} -2 = -k(1+x) \rightarrow k = \frac{2}{1+x} \\ 6-x = 6k \leftarrow E \text{ 代入} \end{cases}$$

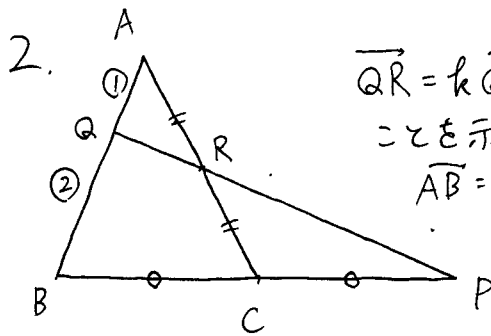
$$6-x = \frac{12}{1+x}$$

$$(6-x)(x+1) = 12$$

$$-x^2 + 5x - 6 = 0$$

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (x-2)(x-3) &= 0 \end{aligned}$$

$$\underline{x = 2, 3}$$



$$\vec{QR} = k\vec{QP} \text{ とする}$$

これを示せばよい.

$$\vec{AB} = \vec{b}, \vec{AC} = \vec{c}$$

とする.

$$\vec{AQ} = \frac{1}{2}\vec{b}, \vec{AR} = \frac{1}{2}\vec{c}, \vec{AP} = -\vec{b} + 2\vec{c}$$

$$\begin{aligned} \vec{QR} &= \vec{AR} - \vec{AQ} = \frac{1}{2}\vec{c} - \frac{1}{2}\vec{b} \\ &= \frac{-2\vec{b} + 3\vec{c}}{2} \dots \textcircled{1} \end{aligned}$$

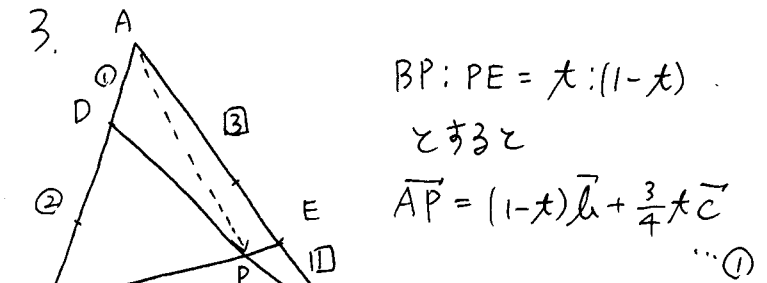
$$\begin{aligned} \vec{QP} &= \vec{AP} - \vec{AQ} = (-\vec{b} + 2\vec{c}) - \frac{1}{2}\vec{b} \\ &= \frac{-4\vec{b} + 6\vec{c}}{2} \dots \textcircled{2} \end{aligned}$$

$$\textcircled{1}, \textcircled{2} \text{ より } \vec{QP} = 4\vec{QR}$$

$$\vec{QR} = \frac{1}{4}\vec{QP}$$

よって P, Q, R は一直線上にある.

$$\underline{QR : QP = 1 : 4}$$



$$BP : PE = t : (1-t)$$

とする

$$\vec{AP} = (1-t)\vec{b} + \frac{3}{4}t\vec{c} \dots \textcircled{1}$$

$$AD = \frac{1}{3}\vec{b}$$

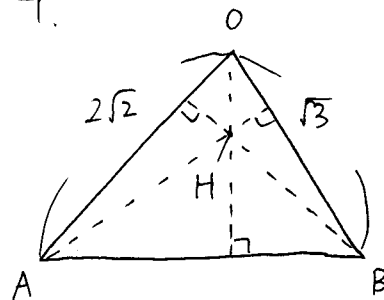
$$\vec{AE} = \frac{3}{4}\vec{c}$$

$$\vec{AP} = \frac{1}{3}s\vec{b} + (1-s)\vec{c} \dots \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \text{ より } \begin{cases} 1-t = \frac{1}{3}s \\ \frac{3}{4}t = 1-s \end{cases}$$

$$\text{これを解いて } \underline{\vec{AP} = \frac{1}{9}\vec{b} + \frac{2}{3}\vec{c}}$$

4.



$$\vec{OA} \cdot \vec{OB} = 2$$

$$\vec{OH} = s\vec{a} + t\vec{b} \text{ とする.}$$

$$\vec{AH} = \vec{OH} - \vec{OA} = (s-1)\vec{a} + t\vec{b}$$

$$\vec{BH} = \vec{OH} - \vec{OB} = s\vec{a} + (t-1)\vec{b}$$

$$\vec{AH} \cdot \vec{OB} = 0 \text{ より}$$

$$((s-1)\vec{a} + t\vec{b}) \cdot \vec{b} = 0$$

$$(s-1)\vec{a} \cdot \vec{b} + t|\vec{b}|^2 = 0$$

$$2(s-1) + 3t = 0 \quad 2s + 3t = 2 \dots \textcircled{1}$$

$$\vec{BH} \cdot \vec{OA} = 0 \text{ より}$$

$$(s\vec{a} + (t-1)\vec{b}) \cdot \vec{a} = 0$$

$$s|\vec{a}|^2 + (t-1)\vec{a} \cdot \vec{b} = 0$$

$$8s + 2(t-1) = 0$$

$$4s + t = 1 \dots \textcircled{2}$$

$$\textcircled{1}, \textcircled{2} \text{ より } s = \frac{1}{10}, t = \frac{3}{5}$$

$$\text{よって } \underline{\vec{OH} = \frac{1}{10}\vec{a} + \frac{3}{5}\vec{b}}$$