(1) $z_2 = \frac{1+\sqrt{3}i}{2} + 1 = \underline{\frac{3+\sqrt{3}i}{2}}$

$z_3 = \frac{1+\sqrt{3}i}{2} \cdot \frac{3+\sqrt{3}i}{2} + 1 = \underline{1+\sqrt{3}i}$

(2) $z_{n+1} - \alpha = \frac{1+\sqrt{3}i}{2}(z_n - \alpha)$ を満たす α を求めると $\alpha = \frac{1+\sqrt{3}i}{2}$

また、$z_1 - \frac{1+\sqrt{3}i}{2} = \frac{1-\sqrt{3}i}{2}$

よって数列 $\left\{z_n - \frac{1+\sqrt{3}i}{2}\right\}$ は初項 $\frac{1-\sqrt{3}i}{2}$、公比 $\frac{1+\sqrt{3}i}{2}$ の等比数列

したがって　$z_n - \frac{1+\sqrt{3}i}{2} = \frac{1-\sqrt{3}i}{2}\left(\frac{1+\sqrt{3}i}{2}\right)^{n-1}$

$$\underline{z_n = \frac{1-\sqrt{3}i}{2}\left(\frac{1+\sqrt{3}i}{2}\right)^{n-1} + \frac{1+\sqrt{3}i}{2}}$$

(3) $-\frac{1-\sqrt{3}i}{2} = \frac{1-\sqrt{3}i}{2}\left(\frac{1+\sqrt{3}i}{2}\right)^{n-1} + \frac{1+\sqrt{3}i}{2}$

$\frac{1-\sqrt{3}i}{2}\left(\frac{1+\sqrt{3}i}{2}\right)^{n-1} = -1$

$\left(\frac{1+\sqrt{3}i}{2}\right)^{n-1} = \frac{-1-\sqrt{3}i}{2}$　　両辺を極形式で表す

$\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{n-1} = \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi$

$\cos\frac{n-1}{3}\pi + i\sin\frac{n-1}{3}\pi = \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi$

したがって、k を0以上の整数とすると

$\frac{n-1}{3}\pi = \frac{4}{3}\pi + 2k\pi$

よって、$n = 6k+5$　$(k = 0, 1, 2, \cdots)$