

$$(1) z^5 = \left(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5} \right)^5$$

$$= \cos 2\pi + i \sin 2\pi = \underline{1}$$

$$(2) z^5 - 1 = 0$$

$$(z-1)(z^4 + z^3 + z^2 + z + 1) = 0$$

$$z \neq 1 \text{ 故) } z^4 + z^3 + z^2 + z + 1 = \underline{0}$$

$$(3) z^2 + \frac{1}{z^2} = \left(z + \frac{1}{z} \right)^2 - 2 = t^2 - 2$$

$$\text{故) } z^2 + z + 1 + \frac{1}{z} + \frac{1}{z^2} = z^2 + \frac{1}{z^2} + z + \frac{1}{z} + 1$$

$$= (t^2 - 2) + t + 1$$

$$= \underline{t^2 + t - 1}$$

$$(4) \frac{1}{z} = z^{-1} = \cos \left(-\frac{2}{5}\pi \right) + i \sin \left(-\frac{2}{5}\pi \right)$$

$$= \cos \frac{2}{5}\pi - i \sin \frac{2}{5}\pi \text{ 故) }$$

$$z + \frac{1}{z} = \left(\cos \frac{2}{5}\pi + i \sin \frac{2}{5}\pi \right) + \left(\cos \frac{2}{5}\pi - i \sin \frac{2}{5}\pi \right)$$

$$= 2 \cos \frac{2}{5}\pi$$

$$(2) \text{ 故) } z^4 + z^3 + z^2 + z + 1 = 0 \quad \left. \begin{array}{l} z^2 \text{ を } t \text{ とする} \\ z^2 + z + 1 + \frac{1}{z} + \frac{1}{z^2} = 0 \end{array} \right\}$$

$$\text{また (3) 故) } t^2 + t - 1 = 0$$

$$\text{これ故) } t = \frac{-1 \pm \sqrt{5}}{2}$$

$$t = 2 \cos \frac{2}{5}\pi > 0 \text{ 故) }$$

$$2 \cos \frac{2}{5}\pi = \frac{-1 + \sqrt{5}}{2}$$

$$\cos \frac{2}{5}\pi = \underline{\underline{\frac{-1 + \sqrt{5}}{4}}}$$