

$$\begin{aligned}
 1. (1-i)^{20} &= \left[\sqrt{2} \left\{ \cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right\} \right]^{20} \\
 &= \sqrt{2}^{20} \left\{ \cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right) \right\}^{20} \\
 &= 2^{10} \left\{ \cos(-5\pi) + i \sin(-5\pi) \right\} \\
 &= 1024 \cdot (-1) = \underline{-1024}
 \end{aligned}$$

$$2. z + \frac{1}{z} = \sqrt{2} \quad \text{よ) } z^2 - \sqrt{2}z + 1 = 0$$

$$z = \frac{1 \pm i}{\sqrt{2}} = \cos\left(\pm \frac{\pi}{4}\right) + i \sin\left(\pm \frac{\pi}{4}\right)$$

$$\begin{aligned}
 \text{よ) } z^{15} + z^{-15} &= \left\{ \cos\left(\pm \frac{\pi}{4}\right) + i \sin\left(\pm \frac{\pi}{4}\right) \right\}^{15} + \left\{ \cos\left(\pm \frac{\pi}{4}\right) + i \sin\left(\pm \frac{\pi}{4}\right) \right\}^{-15} \\
 &= \cos\left(\pm \frac{15}{4}\pi\right) + i \sin\left(\pm \frac{15}{4}\pi\right) + \cos\left(\mp \frac{15}{4}\pi\right) + i \sin\left(\mp \frac{15}{4}\pi\right) \\
 &= \cos \frac{7}{4}\pi + i \sin \frac{7}{4}\pi + \cos \frac{7}{4}\pi - i \sin \frac{7}{4}\pi \\
 &= 2 \cos \frac{7}{4}\pi = \underline{\sqrt{2}}
 \end{aligned}$$

$$3. z = r(\cos \theta + i \sin \theta) \quad \text{よ) } z^2 = r^2(\cos 2\theta + i \sin 2\theta)$$

$$\text{また } 2i = 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$$

$$\text{よ) } 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right) = r^2(\cos 2\theta + i \sin 2\theta)$$

$$\text{よ) } r = \sqrt{2}, \quad \theta = \frac{\pi}{4}, \frac{5}{4}\pi$$

$$\text{よ) } z = \sqrt{2}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) = \underline{1+i}$$

$$z = \sqrt{2}\left(\cos \frac{5}{4}\pi + i \sin \frac{5}{4}\pi\right) = \underline{-1-i}$$

$$4. (1) \frac{\beta - \alpha}{\gamma - \alpha} = 2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right) \quad \text{よ) } \underline{\angle BAC = \frac{\pi}{4}}$$

(2) 絶対値をとると

$$\left| \frac{\beta - \alpha}{\gamma - \alpha} \right| = 2$$

$$|\beta - \alpha| = 2|\gamma - \alpha|$$

$$AB = 2AC$$

よ)

$$\underline{AB:AC = 2:1}$$