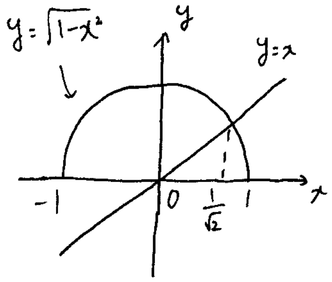
(1) $x \geqq \sqrt{1-x^2}$



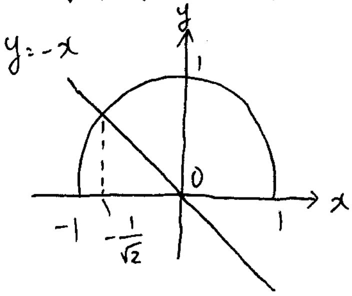
図より

$x \geqq \sqrt{1-x^2}$

を満たすxは

$\underline{\frac{1}{\sqrt{2}} \leqq x \leqq 1}$

(2) $\sqrt{1-x^2} \geqq -x$



図より

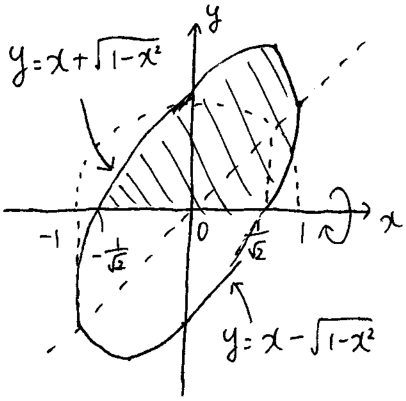
$\sqrt{1-x^2} \geqq -x$

を満たすxは

$\underline{-\frac{1}{\sqrt{2}} \leqq x \leqq 1}$

(3) $y^2-2xy+2x^2-1=0$をxについて解くと

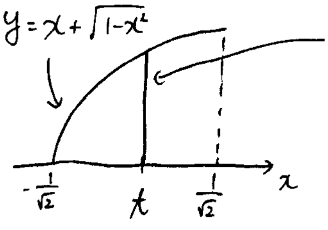
$y = x \pm \sqrt{1-x^2}$



求める立体の形は、図の斜線部分をx軸のまわりに1回転したもの。

(i) $-1 \leqq t < -\frac{1}{\sqrt{2}}$のとき、$\underline{S(t)=0}$

(ii) $-\frac{1}{\sqrt{2}} \leqq t < \frac{1}{\sqrt{2}}$ のとき



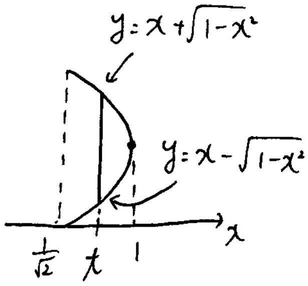
半径は $t+\sqrt{1-t^2}$

よって

$S(t)=\pi\left(t+\sqrt{1-t^2}\right)^2$

$\underline{=\pi\left(1+2t\sqrt{1-t^2}\right)}$

(iii) $\frac{1}{\sqrt{2}} \leqq t \leqq 1$ のとき



切り口はドーナツ型になるので

(全体)−(中身)

を求めればよい。

右上へ

$S(t)=\pi\left(t+\sqrt{1-t^2}\right)^2-\pi\left(t-\sqrt{1-t^2}\right)^2$

$\underline{=4\pi t\sqrt{1-t^2}}$

(4) (3)の(i)、(ii)、(iii)を用いると

$$V=\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}\pi\left(1+2t\sqrt{1-t^2}\right)dt+\int_{\frac{1}{\sqrt{2}}}^{1}4\pi t\sqrt{1-t^2}\,dt$$

$$=\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}\pi\,dt+\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}2\pi t\sqrt{1-t^2}\,dt+\int_{\frac{1}{\sqrt{2}}}^{1}4\pi t\sqrt{1-t^2}\,dt$$

$$=\sqrt{2}\pi+2\pi\underbrace{\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}t\sqrt{1-t^2}\,dt}_{=0}+\underbrace{4\pi\int_{\frac{1}{\sqrt{2}}}^{1}t\sqrt{1-t^2}\,dt}_{(*)}$$

(*)について

$1-t^2=u$とおくと$\frac{du}{dt}=-2t$

t	$\frac{1}{\sqrt{2}} \to 1$
u	$\frac{1}{2} \to 0$

$$(*)=4\pi\cdot\int_{\frac{1}{2}}^{0}\sqrt{u}\cdot\left(-\frac{1}{2}\right)du$$

$$=2\pi\int_{0}^{\frac{1}{2}}\sqrt{u}\,du$$

$$=2\pi\left[\frac{2}{3}u^{\frac{3}{2}}\right]_0^{\frac{1}{2}}$$

$$=2\pi\cdot\frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{3}\pi$$

よって、

$$V=\sqrt{2}\pi+\frac{\sqrt{2}}{3}\pi=\underline{\frac{4\sqrt{2}}{3}\pi}$$