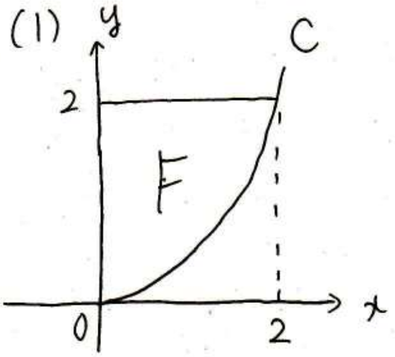
(1)



$y=2$ のときの $y=\frac{1}{2}x+\frac{1}{8}x^3$ の x座標は

$$2=\frac{1}{2}x+\frac{1}{8}x^3$$

$$x^3+4x-16=0$$

$$(x-2)(x^2+2x+8)=0$$

$$x=2$$

よって F の面積は

$$S=4-\int_0^2\left(\frac{1}{2}x+\frac{1}{8}x^3\right)dx$$

$$=4-\left[\frac{1}{4}x^2+\frac{1}{32}x^4\right]_0^2=4-\frac{3}{2}=\underline{\frac{5}{2}}$$

(2) $g^{-1}(2)=a$ とおくと $g(a)=2 \iff f^{-1}(a)^2=2$

$$\iff f^{-1}(a)=\sqrt{2}$$

$$\iff a=f(\sqrt{2})$$

よって $a=f(\sqrt{2})=\frac{\sqrt{2}}{2}+\frac{2\sqrt{2}}{8}=\underline{\frac{3\sqrt{2}}{4}}$

(3) 求める体積をバームクーヘン分割を用いて求めると

$$V=4\pi\cdot 2-\int_0^2 2\pi x\left(\frac{1}{2}x+\frac{1}{8}x^3\right)dx$$

$$=8\pi-2\pi\int_0^2\left(\frac{1}{2}x^2+\frac{1}{8}x^4\right)dx$$

$$=8\pi-2\pi\left[\frac{1}{6}x^3+\frac{1}{40}x^5\right]_0^2$$

$$=8\pi-\frac{64}{15}\pi=\underline{\frac{56}{15}\pi}$$