(1) $f(x) = \int_1^x \log t\, dt + a$

$= x\log x - x + 1 + a$

$a = \int_1^3 (t\log t - t + 1 + a)\, dt$

$= \left[ \frac{1}{2}t^2 \log t - \frac{3}{4}t^2 + t + at \right]_1^3$

$a = \frac{9}{2}\log 3 - 4 + 2a$

よって

$\underline{a = 4 - \frac{9}{2}\log 3}$

$\underline{f(x) = x\log x - x + 5 - \frac{9}{2}\log 3}$

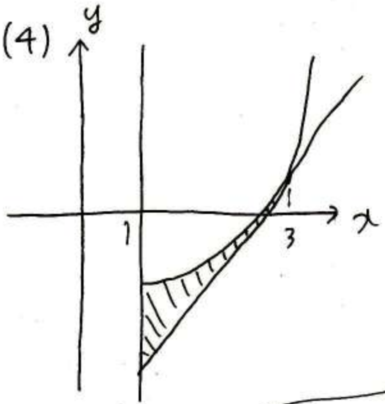
(2) $f'(x) = \log x$, $f''(x) = \frac{1}{x}$　$x \geqq 1$ より <u>下に凸</u>

(3) $f(3) = 2 - \frac{3}{2}\log 3$, $f'(3) = \log 3$ より接線の方程式は

$y - (2 - \frac{2}{3}\log 3) = \log 3 \cdot (x - 3)$

よって　$\underline{y = x\log 3 + 2 - \frac{9}{2}\log 3}$

(4)




求める面積は右図の斜線部分

したがって

$S = \int_1^3 f(x)\,dx - \int_1^3 (x\log 3 + 2 - \frac{9}{2}\log 3)$

$= a - \left[ \frac{1}{2}x^2 \log 3 + 2x - \frac{9}{2}x\log 3 \right]_1^3$

$= (4 - \frac{9}{2}\log 3) - \left\{ \left( \frac{9}{2}\log 3 + 6 - \frac{27}{2}\log 3 \right) - \left( \frac{1}{2}\log 3 + 2 - \frac{9}{2}\log 3 \right) \right\}$

$= \underline{\frac{1}{2}\log 3}$