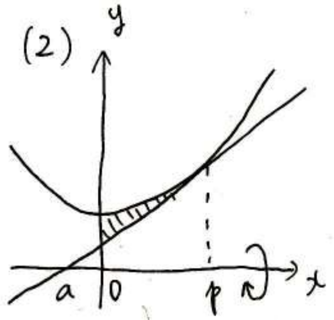
(1) $y' = \dfrac{x}{\sqrt{x^2+1}}$ より接線の方程式は $y - \sqrt{p^2+1} = \dfrac{p}{\sqrt{p^2+1}}(x-p)$

よって、 $\underline{y = \dfrac{p}{\sqrt{p^2+1}}x + \dfrac{1}{\sqrt{p^2+1}}}$

この式に $y=0$ を代入して、 $0 = \dfrac{p}{\sqrt{p^2+1}}x + \dfrac{1}{\sqrt{p^2+1}}$

$x = -\dfrac{1}{p}$ よって、 $\underline{a = -\dfrac{1}{p}}$

(2)



求める体積は図の斜線部分を x 軸のまわりに1回転したもの

$$V = \pi\int_0^p (x^2+1)\,dx - \pi\int_0^p \left(\frac{p}{\sqrt{p^2+1}}x + \frac{1}{\sqrt{p^2+1}}\right)^2 dx$$

$$= \pi\left(\frac{1}{3}p^3 + p\right) - \frac{\pi}{3}\left(p^3 + 2p + \frac{p}{p^2+1}\right)$$

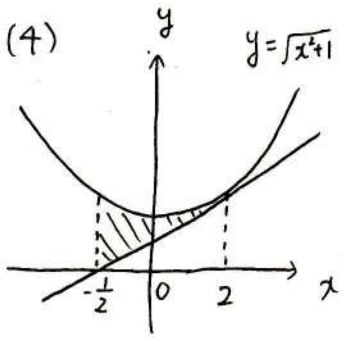
$$= \frac{1}{3}p - \frac{p}{3(p^2+1)} = \underline{\frac{p^3\pi}{3(p^2+1)}}$$

(3) $f'(x) = \sqrt{x^2+1} + x\cdot\dfrac{x}{\sqrt{x^2+1}} + \dfrac{1}{x+\sqrt{x^2+1}}\left(1 + \dfrac{x}{\sqrt{x^2+1}}\right)$

$$= \sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+1}}$$

$$= \frac{(x^2+1) + x^2+1}{\sqrt{x^2+1}} = \frac{2(x^2+1)}{\sqrt{x^2+1}} = \underline{2\sqrt{x^2+1}}$$

(4)



求める面積は図の斜線部分

下の三角形の面積

よって $S = \displaystyle\int_{-\frac{1}{2}}^{2} \sqrt{x^2+1}\,dx - \frac{5}{2}\cdot\sqrt{5}\cdot\frac{1}{2}$

$$= \frac{1}{2}\Big[x\sqrt{x^2+1} + \log(x+\sqrt{x^2+1})\Big]_{-\frac{1}{2}}^{2} - \frac{5\sqrt{5}}{4}$$

$$= \frac{9\sqrt{5}}{8} + \frac{1}{2}\log\frac{7+3\sqrt{5}}{2} - \frac{5\sqrt{5}}{4}$$

$$= \underline{-\frac{\sqrt{5}}{8} + \frac{1}{2}\log\frac{7+3\sqrt{5}}{2}}$$