1. (1) 左辺 $= \sin x$

$= 2\sin\frac{x}{2}\cos\frac{x}{2}$

$= 2\tan\frac{x}{2}\cdot\cos^2\frac{x}{2}$

$= \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \dfrac{2t}{1+t^2} =$ 右辺

(2) 左辺 $= \cos x$

$= \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$

$= \cos^2\frac{x}{2}\left(1-\tan^2\frac{x}{2}\right)$

$= \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \dfrac{1-t^2}{1+t^2} =$ 右辺

(3) 左辺 $= \tan x$

$= \dfrac{\sin x}{\cos x} = \dfrac{2t}{1-t^2} =$ 右辺

2.

$$\int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x + 1} \quad \cdots (*)$$

$t = \tan\frac{x}{2}$ とすると

$\dfrac{dt}{dx} = \dfrac{1}{\cos^2\frac{x}{2}}\cdot\dfrac{1}{2}$

$dx = 2\cos^2\frac{x}{2}\,dt$

$dx = \dfrac{2}{1+t^2}\,dt$

x	$0 \to \frac{\pi}{2}$
t	$0 \to 1$

よって

$$(*) = \int_0^1 \frac{1}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} + 1}\cdot\frac{2}{1+t^2}\,dt$$

$$= \int_0^1 \frac{2}{2t + (1-t^2) + (1+t^2)}\,dt$$

$$= \int_0^1 \frac{dt}{t+1} = \Big[\log(t+1)\Big]_0^1$$

$$= \underline{\log 2}$$

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$$\int_0^{\frac{\pi}{2}} \frac{dx}{3\sin x + 4\cos x + 5}$$

$t = \tan\frac{x}{2}$ と置換

$$= \int_0^1 \frac{1}{3\cdot\frac{2t}{1+t^2} + 4\cdot\frac{1-t^2}{1+t^2} + 5}\cdot\frac{2}{1+t^2}\,dt$$

$$= \int_0^1 \frac{2}{6t + 4(1-t^2) + 5(1+t^2)}\,dt$$

$$= \int_0^1 \frac{2}{t^2 + 6t + 9}\,dt$$

$$= 2\int_0^1 \frac{dt}{(t+3)^2}$$

$$= 2\left[-\frac{1}{t+3}\right]_0^1$$

$$= 2\left(-\frac{1}{4} + \frac{1}{3}\right) = \underline{\frac{1}{6}}$$