(1) $f'(x) = \underline{1 - 2\sin x}$

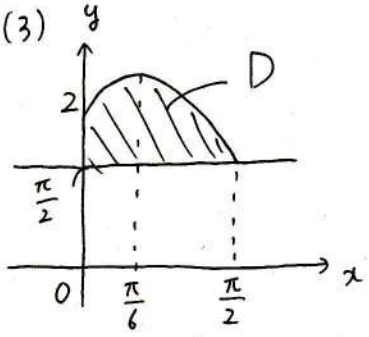
(2) $f'(x) = 0$ となる x は、$0 \leqq x \leqq \pi$ では $x = \frac{\pi}{6}$、$\frac{5}{6}\pi$

x	0	$\cdots$	$\frac{\pi}{6}$	$\cdots$	$\frac{5}{6}\pi$	$\cdots$	π
$f'(x)$	/	+	0	−	0	+	/
$f(x)$	2	↗	$\frac{\pi}{6}+\sqrt{3}$	↘	$\frac{5}{6}\pi-\sqrt{3}$	↗	$\pi-2$

よって
$M = f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \sqrt{3}$

$\underline{m = f\left(\frac{5}{6}\pi\right) = \frac{5}{6}\pi - \sqrt{3}}$

(3)



求める面積は左図の斜線部分

したがって $S = \int_0^{\frac{\pi}{2}} \left(x + 2\cos x - \frac{\pi}{2}\right) dx$

$= \left[\frac{1}{2}x^2 + 2\sin x - \frac{\pi}{2}x\right]_0^{\frac{\pi}{2}}$

$= \underline{2 - \frac{\pi^2}{8}}$

(4) $I = \int x\cos x\, dx$

$= \int x (\sin x)' dx$

$= x\sin x - \int \sin x\, dx = \underline{x\sin x + \cos x + C}$

(5) 求める体積を V とすると

くりぬく円柱の体積

$V = \pi \int_0^{\frac{\pi}{2}} (x + 2\cos x)^2 dx - \frac{\pi}{2} \cdot \frac{\pi}{2} \cdot \pi \cdot \frac{\pi}{2}$

$= \pi \int_0^{\frac{\pi}{2}} (x^2 + 4x\cos x + 4\cos^2 x) dx - \frac{\pi^4}{8}$

$= \pi \int_0^{\frac{\pi}{2}} (x^2 + 4x\cos x + 2 + 2\cos 2x) dx - \frac{\pi^4}{8}$

$= \pi \left[\frac{1}{3}x^3 + 4(x\sin x + \cos x) + 2x + \sin 2x\right]_0^{\frac{\pi}{2}} - \frac{\pi^4}{8}$

$= \frac{\pi^4}{24} + 3\pi^2 - 4\pi - \frac{\pi^4}{8}$

$= \underline{-\frac{\pi^4}{12} + 3\pi^2 - 4\pi}$