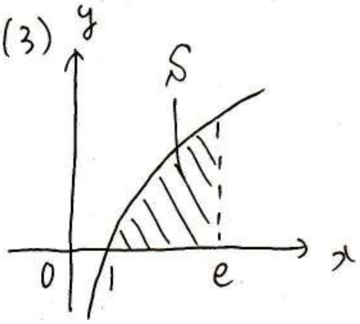
(1) $f(x) = \dfrac{\log x}{\sqrt{x}}$

$$f'(x) = \frac{\frac{1}{x}\cdot\sqrt{x} - \log x \cdot \frac{1}{2\sqrt{x}}}{x} = \underline{\frac{2-\log x}{2x\sqrt{x}}}$$

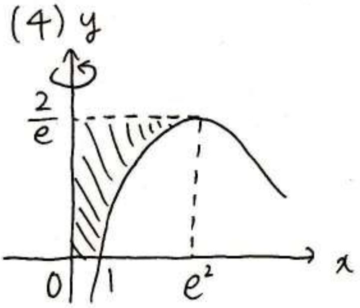
(2)
$$\begin{aligned}\int \frac{\log x}{\sqrt{x}}\,dx &= \int (2\sqrt{x})' \log x\,dx \\ &= 2\sqrt{x}\log x - \int 2\sqrt{x}\cdot\frac{1}{x}\,dx \\ &= 2\sqrt{x}\log x - 2\int\frac{dx}{\sqrt{x}} \\ &= \underline{2\sqrt{x}\log x - 4\sqrt{x} + C}\end{aligned}$$

(3)



$$\begin{aligned}S &= \int_1^e \frac{\log x}{\sqrt{x}}\,dx \\ &= \Big[2\sqrt{x}\log x - 4\sqrt{x}\Big]_1^e \\ &= \underline{4 - 2\sqrt{e}}\end{aligned}$$

(4)



$f'(x) = \dfrac{2-\log x}{2x\sqrt{x}}$ より $\alpha = e^2$

したがって求める体積は左図の斜線部分をy軸のまわりに1回転させたものである.

よって $V = \underbrace{\pi e^4 \cdot \frac{2}{e}}_{\text{全体の円柱}} - \underbrace{\int_1^{e^2} 2\pi x \frac{\log x}{\sqrt{x}}\,dx}_{\text{いらない部分(バームクーヘン)}}$

$\left(\frac{2}{3}\sqrt{x^3}\right)'$

$$\begin{aligned}V &= 2\pi e^3 - 2\pi\int_1^{e^2}\sqrt{x}\log x\,dx \\ &= 2\pi e^3 - 2\pi\left(\Big[\frac{2}{3}x\sqrt{x}\log x\Big]_1^{e^2} - \int_1^{e^2}\frac{2}{3}\sqrt{x}\,dx\right) \\ &= 2\pi e^3 - 2\pi\Big[\frac{2}{3}x\sqrt{x}\log x - \frac{4}{9}x\sqrt{x}\Big]_1^{e^2} \\ &= 2\pi e^3 - 2\pi\left(\frac{4}{3}e^3 - \frac{4}{9}e^3 + \frac{4}{9}\right) \\ &= \underline{\frac{2}{9}\pi(e^3 - 4)}\end{aligned}$$