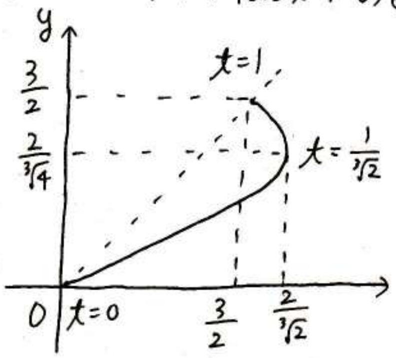
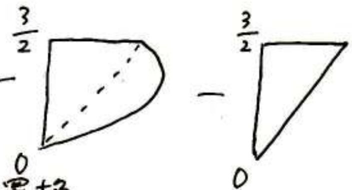
(1) $\dfrac{dx}{dt} = \dfrac{3(1-2t^3)}{(1+t^3)^2}$、$\dfrac{dy}{dt} = \dfrac{3t(2-t^3)}{(1+t^3)^2}$ より.

t	0	$\cdots$	$\frac{1}{\sqrt[3]{2}}$	$\cdots$	1
$\frac{dx}{dt}$	/	$+$	0	$-$	/
x	0	$\rightarrow$	$\frac{2}{\sqrt[3]{2}}$	$\leftarrow$	$\frac{3}{2}$
$\frac{dy}{dt}$	/	$+$	$+$	$+$	/
y	0	$\uparrow$	$\frac{2}{\sqrt[3]{4}}$	$\uparrow$	$\frac{3}{2}$
(x,y)	$(0,0)$	$\nearrow$	$\left(\frac{2}{\sqrt[3]{2}}, \frac{2}{\sqrt[3]{4}}\right)$	$\nwarrow$	$\left(\frac{3}{2}, \frac{3}{2}\right)$

したがってグラフは以下のようになる



(2) 求める面積を S とすると



$$S = \int_0^{\frac{3}{2}} x\,dy - \frac{3}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}$$

tで置換

y	$0 \to \frac{3}{2}$
t	$0 \to 1$

$$= \int_0^1 \frac{3t}{1+t^3}\cdot\frac{3t(2-t^3)}{(1+t^3)^2}dt - \frac{9}{8}$$

$$= \int_0^1 \frac{9t^2(2-t^3)}{(1+t^3)^3}dt - \frac{9}{8} = (*) \qquad 2-t^3 = 3-u$$

ここで $1+t^3 = u$ とおくと $\dfrac{du}{dt} = 3t^2 \Rightarrow du = 3t^2 dt$

t	$0 \to 1$
u	$1 \to 2$

$$(*) = \int_1^2 \frac{3(3-u)}{u^3}du - \frac{9}{8}$$

$$= 3\int_1^2 \left(\frac{3}{u^3} - \frac{1}{u^2}\right)du - \frac{9}{8}$$

$$= 3\left[-\frac{3}{2u^2} + \frac{1}{u}\right]_1^2 - \frac{9}{8}$$

$$= 3\cdot\frac{5}{8} - \frac{9}{8} = \underline{\frac{3}{4}}$$