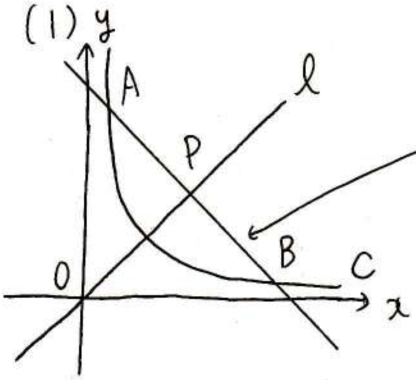
(1)



$OP = t$ とおくと点Pは $\left(\frac{1}{\sqrt{2}}t, \frac{1}{\sqrt{2}}t\right)$ となる

$y = -x + k$ が点Pを通るので

$\frac{1}{\sqrt{2}}t = -\frac{1}{\sqrt{2}}t + k \Rightarrow k = \sqrt{2}t$

$y = \frac{1}{x}$ と $y = -x + \sqrt{2}t$ を連立させると

$\frac{1}{x} = -x + \sqrt{2}t$

$x^2 - \sqrt{2}tx + 1 = 0 \cdots (*)$

$(*)$ の解を α, β $(\alpha < \beta)$ とおくと

$\alpha + \beta = \sqrt{2}t, \quad \alpha\beta = 1$

また、$A\left(\alpha, \frac{1}{\alpha}\right)$, $B\left(\beta, \frac{1}{\beta}\right)$

PはABの中点なので $AP = \frac{1}{2}AB$ より $AP^2 = \frac{1}{4}AB^2$

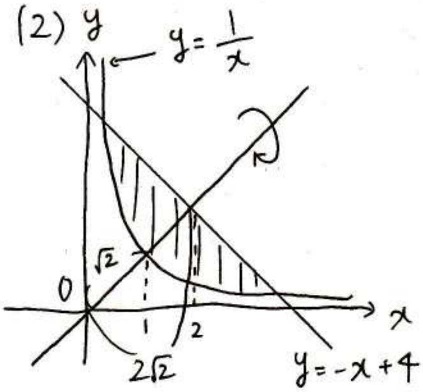
よって $AP^2 = \frac{1}{4}\left\{(\alpha-\beta)^2 + \left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2\right\}$

$= \frac{1}{4}\left\{(\alpha^2 - 2\alpha\beta + \beta^2) + \left(\frac{1}{\alpha^2} - \frac{2}{\alpha\beta} + \frac{1}{\beta^2}\right)\right\}$

$= \frac{1}{4}\left\{(\alpha+\beta)^2 - 4\alpha\beta + \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2} - \frac{2}{\alpha\beta}\right\}$

$= t^2 - 2$ 　したがって $\underline{AP = \sqrt{t^2 - 2}}$

(2)



求める体積は

$V = \pi \int_{\sqrt{2}}^{2\sqrt{2}} AP^2 dt$

$= \pi \int_{\sqrt{2}}^{2\sqrt{2}} (t^2 - 2) dt$

$= \pi \left[\frac{1}{3}t^3 - 2t\right]_{\sqrt{2}}^{2\sqrt{2}}$

$= \underline{\frac{8\sqrt{2}}{3}\pi}$