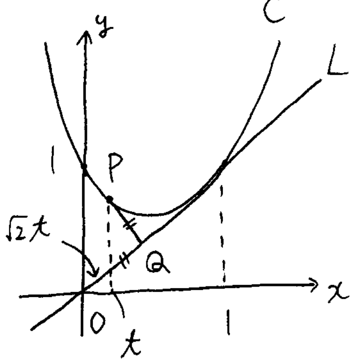
(1). (2).



点 $P(t,\ t^2-t+1)$ と $L:\ y=x$ の距離 PQ は

$$PQ=\frac{|t-(t^2-t+1)|}{\sqrt{2}}$$

$$=\frac{|-(t-1)^2|}{\sqrt{2}}=\frac{(t-1)^2}{\sqrt{2}}$$

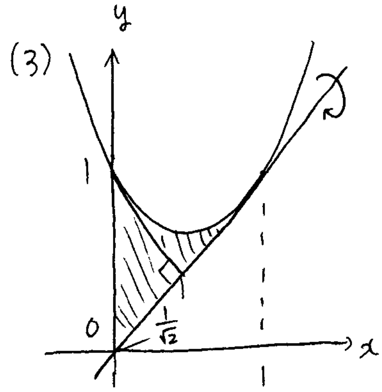
よって $\underline{PQ=\dfrac{(t-1)^2}{\sqrt{2}}}$

$$u=OQ=\sqrt{2}\,t+\frac{(t-1)^2}{\sqrt{2}}$$

$$=\frac{2t+(t-1)^2}{\sqrt{2}}$$

$$=\underline{\frac{t^2+1}{\sqrt{2}}}$$

(3)



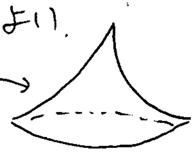
求める立体の体積は図の斜線部分を $y=x$ の周りに1回転したもの.

0から $\frac{1}{\sqrt{2}}$ までで回転させてできた立体は円錐なので,

$$\left(\frac{1}{\sqrt{2}}\right)^2\cdot\pi\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{3}=\frac{\sqrt{2}}{12}\pi$$

残りの上側の立体の体積を求めればよい.

こんな感じのもの



右上へ

上側の体積は

$$V=\pi\int_{\frac{1}{\sqrt{2}}}^{\sqrt{2}}PQ^2\,du$$

$u=\dfrac{t^2+1}{\sqrt{2}}$ より $\dfrac{du}{dt}=\sqrt{2}\,t$

$$\begin{array}{c|c} u & \frac{1}{\sqrt{2}}\to\sqrt{2} \\ \hline t & 0\to 1 \end{array}$$

$$=\pi\int_0^1\left(\frac{(t-1)^2}{\sqrt{2}}\right)^2\cdot\sqrt{2}\,t\,dt$$

$$=\frac{\sqrt{2}}{2}\pi\underline{\int_0^1(t-1)^4\cdot t\,dt}\quad(*)$$

$$(*)=\int_0^1\left(\frac{1}{5}(t-1)^5\right)'\cdot t\,dt$$

$$=\left[\frac{1}{5}(t-1)^5t\right]_0^1-\int_0^1\frac{1}{5}(t-1)^5\,dt$$

$$=-\frac{1}{30}\left[(t-1)^6\right]_0^1=\frac{1}{30}$$

よって

$$V=\frac{\sqrt{2}}{2}\pi\cdot\frac{1}{30}=\frac{\sqrt{2}}{60}\pi$$

したがって全体の体積は

$$\frac{\sqrt{2}}{12}\pi+\frac{\sqrt{2}}{60}\pi=\underline{\frac{\sqrt{2}}{10}\pi}$$