(1) $f'(x) = \dfrac{1-\log x}{x^2}$

(2) 接点のx座標をtとする.
接線の方程式は

$$y - \frac{\log t}{t} = \frac{1-\log t}{t^2}(x-t)$$

この式が原点$(0,0)$を通るので

$$-\frac{\log t}{t} = -\frac{1-\log t}{t}$$

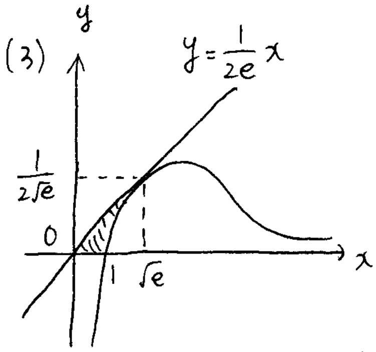
$\log t = \dfrac{1}{2}$　よって　$t = \sqrt{e}$

したがって

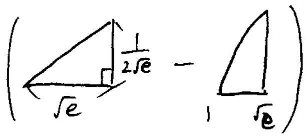
$$y - \frac{1}{2\sqrt{e}} = \frac{1}{2e}(x-\sqrt{e})$$

$$\underline{y = \frac{1}{2e}x}$$

(3)



求める面積は
図の斜線部分



よって　$S = \sqrt{e}\cdot\dfrac{1}{2\sqrt{e}}\cdot\dfrac{1}{2} - \displaystyle\int_1^{\sqrt{e}}\frac{\log x}{x}dx$

$$= \frac{1}{4} - \underbrace{\int_1^{\sqrt{e}}\frac{\log x}{x}dx}_{=(*)}$$

$(*)$について　$t = \log x$とおくと

$\dfrac{dt}{dx} = \dfrac{1}{x}$.

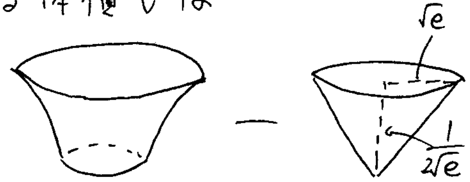
x	$1 \to \sqrt{e}$
t	$0 \to \frac{1}{2}$

$$(*) = \int_0^{\frac{1}{2}} t\,dt = \left[\frac{1}{2}t^2\right]_0^{\frac{1}{2}} = \frac{1}{8}.$$

よって.

$$S = \frac{1}{4} - \frac{1}{8} = \underline{\frac{1}{8}}$$

(4) 求める体積Vは



となるので

$$V = \pi\int_0^{\frac{1}{2\sqrt{e}}} x^2dy - \pi e\cdot\frac{1}{2\sqrt{e}}\cdot\frac{1}{3}$$

$$= \pi\underbrace{\int_0^{\frac{1}{2\sqrt{e}}} x^2dy}_{(*)} - \frac{\sqrt{e}}{6}\pi$$

$(*)$について　$y = \dfrac{\log x}{x}$とすると

$\dfrac{dy}{dx} = \dfrac{1-\log x}{x^2}$.

y	$0 \to \frac{1}{2\sqrt{e}}$
x	$1 \to \sqrt{e}$

よって

$$(*) = \int_1^{\sqrt{e}} x^2\cdot\frac{1-\log x}{x^2}dx$$

$$= \int_1^{\sqrt{e}}(1-\log x)dx$$

$$= \Big[x-(x\log x - x)\Big]_1^{\sqrt{e}}$$

$$= \Big[2x - x\log x\Big]_1^{\sqrt{e}}$$

$$= \left(2\sqrt{e} - \frac{1}{2}\sqrt{e}\right) - (2-0)$$

$$= \frac{3}{2}\sqrt{e} - 2$$

したがって.

$$V = \left(\frac{3}{2}\sqrt{e}-2\right)\pi - \frac{\sqrt{e}}{6}\pi$$

$$= \underline{\frac{4}{3}\sqrt{e}\pi - 2\pi}$$