(1) $-\log 2 = \log(1-x^2)$ より

$$1-x^2=\frac{1}{2}$$

$$x^2=\frac{1}{2} \qquad \underline{x=\pm\frac{1}{\sqrt{2}}}$$

(2) $\underline{f'(x)=\dfrac{-2x}{1-x^2}}$

(3)

x	-1	$\cdots$	0	$\cdots$	1
$f'(x)$	/	$+$	0	$-$	/
$f(x)$	/	$\nearrow$	$\log 2$	$\searrow$	/

したがって、$\underline{x=0\text{で極大値 }\log 2}$

(4)

$$I(t)=\int_0^t \frac{dx}{1-x^2}$$

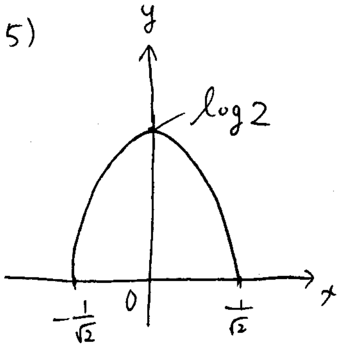
$$=\frac{1}{2}\int_0^t\left(\frac{1}{1+x}+\frac{1}{1-x}\right)dx$$

$$=\frac{1}{2}\Big[\log(1+x)-\log(1-x)\Big]_0^t$$

$$=\frac{1}{2}\left[\log\frac{1+x}{1-x}\right]_0^t$$

$$\underline{=\frac{1}{2}\log\frac{1+t}{1-t}}$$

(5)



求める曲線の長さは図の部分。

$$L=\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}\sqrt{1+\{f'(x)\}^2}\,dx$$

$1+\{f'(x)\}^2$ を計算すると

$$1+\{f'(x)\}^2=1+\left(\frac{-2x}{1-x^2}\right)^2$$

$$=1+\frac{4x^2}{(1-x^2)^2}$$

$$=\frac{(1-x^2)^2+4x^2}{(1-x^2)^2}$$

$$=\frac{1-2x^2+x^4+4x^2}{(1-x^2)^2}$$

$$=\frac{1+2x^2+x^4}{(1-x^2)^2}=\frac{(1+x^2)^2}{(1-x^2)^2}$$

よって

$$L=\int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}\frac{1+x^2}{1-x^2}\,dx$$

$$=2\int_0^{\frac{1}{\sqrt{2}}}\frac{1+x^2}{1-x^2}\,dx$$

$$=2\int_0^{\frac{1}{\sqrt{2}}}\left(\frac{2}{1-x^2}-1\right)dx$$

$$=4\cdot\frac{1}{2}\log\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}-2\cdot\frac{1}{\sqrt{2}}$$

$$=2\log\frac{\sqrt{2}+1}{\sqrt{2}-1}-\sqrt{2}$$

$$=2\log(\sqrt{2}+1)^2-\sqrt{2}$$

$$\underline{=4\log(\sqrt{2}+1)-\sqrt{2}}$$