(1) $x = \frac{2}{3}t^3$、$y = t^2$ $(0 \leqq t \leqq 1)$

$$\frac{dx}{dt} = 2t^2、\frac{dy}{dt} = 2t$$

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{4t^4 + 4t^2}$$
$$= \sqrt{4t^2(t^2+1)}$$
$$= 2t\sqrt{t^2+1}$$

よって曲線の長さ L は

$$L = \int_0^1 2t\sqrt{t^2+1}\,dt = (*)$$

$t^2+1 = u$ とおくと $\frac{du}{dt} = 2t$、

t	$0 \to 1$
u	$1 \to 2$

$$(*) = \int_1^2 \sqrt{u}\,du$$
$$= \left[\frac{2}{3}u\sqrt{u}\right]_1^2$$
$$= \underline{\frac{2}{3}(2\sqrt{2} - 1)}$$

(2) $x = e^\theta \cos\theta$、$y = e^\theta \sin\theta$ $(0 \leqq \theta \leqq \pi)$

$$\frac{dx}{d\theta} = e^\theta \cos\theta - e^\theta \sin\theta$$
$$= e^\theta(\cos\theta - \sin\theta)$$
$$\frac{dy}{d\theta} = e^\theta \sin\theta + e^\theta \cos\theta$$
$$= e^\theta(\sin\theta + \cos\theta)$$

$$\sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} = \sqrt{e^{2\theta}(\cos\theta - \sin\theta)^2 + e^{2\theta}(\sin\theta + \cos\theta)^2}$$
$$= \sqrt{2}\,e^\theta$$

よって曲線の長さ L は

$$L = \sqrt{2}\int_0^\pi e^\theta\,d\theta$$
$$= \sqrt{2}\left[e^\theta\right]_0^\pi$$
$$= \underline{\sqrt{2}(e^\pi - 1)}$$

(3) $y = \sqrt{4 - x^2}$ $(-1 \leqq x \leqq 1)$

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{\sqrt{4-x^2}} \cdot (-2x)$$
$$= -\frac{x}{\sqrt{4-x^2}}$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{x^2}{4-x^2}}$$
$$= \sqrt{\frac{4}{4-x^2}} = \frac{2}{\sqrt{4-x^2}}$$

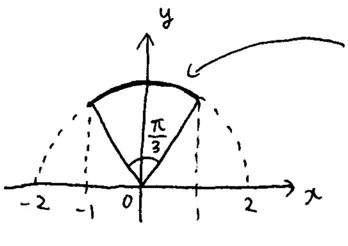
よって曲線の長さ L は

$$L = 2\int_{-1}^1 \frac{dx}{\sqrt{4-x^2}}$$
$$= 2\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2\cos\theta} \cdot 2\cos\theta\,d\theta$$
$$= 2\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} d\theta = \underline{\frac{2}{3}\pi}$$

$x = 2\sin\theta$

$\frac{dx}{d\theta} = 2\cos\theta$

x	$-1 \to 1$
θ	$-\frac{\pi}{6} \to \frac{\pi}{6}$



問題の曲線の長さは半径2、中心角 $\frac{\pi}{3}$ の扇形の弧の長さなので

$$2 \times \frac{\pi}{3} = \underline{\frac{2}{3}\pi}$$

(4) $y = \log(1 - x^2)$ $\left(0 \leqq x \leqq \frac{1}{2}\right)$

$$\frac{dy}{dx} = -\frac{2x}{1-x^2}$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \frac{4x^2}{(1-x^2)^2}}$$
$$= \sqrt{\frac{(1-x^2)^2 + 4x^2}{(1-x^2)^2}}$$
$$= \sqrt{\frac{(1+x^2)^2}{(1-x^2)^2}}$$
$$= \frac{1+x^2}{1-x^2} = \frac{1}{1+x} + \frac{1}{1-x} - 1$$

$$\frac{2-(1-x^2)}{1-x^2} = \frac{2}{1-x^2} - 1$$

よって曲線の長さ L は

$$L = \int_0^{\frac{1}{2}} \left(\frac{1}{1+x} + \frac{1}{1-x} - 1\right)dx$$
$$= \left[\log\frac{1+x}{1-x} - x\right]_0^{\frac{1}{2}}$$
$$= \underline{\log 3 - \frac{1}{2}}$$

(5) $y=\log(\cos x) \quad \left(0 \leqq x \leqq \frac{\pi}{3}\right)$

$$\frac{dy}{dx}=\frac{-\sin x}{\cos x}=-\tan x$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2}=\sqrt{1+\tan^2 x}=\sqrt{\frac{1}{\cos^2 x}}=\frac{1}{\cos x}$$

よって曲線の長さLは

$$L=\int_0^{\frac{\pi}{3}}\frac{dx}{\cos x}$$

$$=\int_0^{\frac{\pi}{3}}\frac{\cos x}{1-\sin^2 x}dx=(*)$$

$\sin x=t$ とおくと $\frac{dt}{dx}=\cos x$

x	$0 \to \frac{\pi}{3}$
t	$0 \to \frac{\sqrt{3}}{2}$

$$(*)=\int_0^{\frac{\sqrt{3}}{2}}\frac{dt}{1-t^2}$$

$$=\frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}}\left(\frac{1}{1+t}+\frac{1}{1-t}\right)dt$$

$$=\frac{1}{2}\Big[\log|1+t|-\log|1-t|\Big]_0^{\frac{\sqrt{3}}{2}}$$

$$=\frac{1}{2}\left[\log\frac{1+t}{1-t}\right]_0^{\frac{\sqrt{3}}{2}}$$

$$=\frac{1}{2}\log\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}$$

$$=\frac{1}{2}\log\frac{2+\sqrt{3}}{2-\sqrt{3}}$$

分母分子に $2+\sqrt{3}$ をかける

$$=\frac{1}{2}\log(2+\sqrt{3})^2$$

$$=\underline{\log(2+\sqrt{3})}$$