

$$(1) x = \frac{2}{3}t^3, y = t^2 \quad (0 \leq t \leq 1)$$

$$\frac{dx}{dt} = 2t^2, \quad \frac{dy}{dt} = 2t$$

$$\begin{aligned} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} &= \sqrt{4t^4 + 4t^2} \\ &= \sqrt{4t^2(t^2+1)} \\ &= 2t\sqrt{t^2+1} \end{aligned}$$

よて曲線の長さLは

$$L = \int_0^1 2t\sqrt{t^2+1} dt = (*)$$

$$t^2+1 = u \text{ とおくと } \frac{du}{dt} = 2t, \quad \left. \begin{array}{l} t \mid 0 \rightarrow 1 \\ u \mid 1 \rightarrow 2 \end{array} \right\}$$

$$\begin{aligned} (*) &= \int_1^2 \sqrt{u} du \\ &= \left[\frac{2}{3} u\sqrt{u} \right]_1^2 \\ &= \frac{2}{3} (2\sqrt{2} - 1) \end{aligned}$$

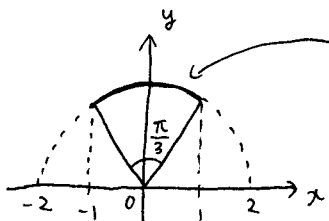
$$(3) y = \sqrt{4-x^2} \quad (-1 \leq x \leq 1)$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2} \cdot \frac{1}{\sqrt{4-x^2}} \cdot (-2x) \\ &= -\frac{x}{\sqrt{4-x^2}} \end{aligned}$$

$$\begin{aligned} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} &= \sqrt{1 + \frac{x^2}{4-x^2}} \\ &= \sqrt{\frac{4}{4-x^2}} = \frac{2}{\sqrt{4-x^2}} \end{aligned}$$

よて曲線の長さLは

$$\begin{aligned} L &= 2 \int_{-1}^1 \frac{dx}{\sqrt{4-x^2}} \quad \left. \begin{array}{l} x = 2 \sin \theta \\ \frac{dx}{d\theta} = 2 \cos \theta \\ \left. \begin{array}{l} x \mid -1 \rightarrow 1 \\ \theta \mid -\frac{\pi}{6} \rightarrow \frac{\pi}{6} \end{array} \right\} \end{array} \right\} \\ &= 2 \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2 \cos \theta} \cdot 2 \cos \theta d\theta \\ &= 2 \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} d\theta = \frac{2}{3} \pi \end{aligned}$$



問題の曲線の長さは
半径2, 中心角 $\frac{\pi}{3}$ の扇形の
弧の長さなので
 $2 \times \frac{\pi}{3} = \frac{2}{3} \pi$

$$(2) x = e^\theta \cos \theta, y = e^\theta \sin \theta \quad (0 \leq \theta \leq \pi)$$

$$\begin{aligned} \frac{dx}{d\theta} &= e^\theta \cos \theta - e^\theta \sin \theta \\ &= e^\theta (\cos \theta - \sin \theta) \end{aligned}$$

$$\begin{aligned} \frac{dy}{d\theta} &= e^\theta \sin \theta + e^\theta \cos \theta \\ &= e^\theta (\sin \theta + \cos \theta) \end{aligned}$$

$$\begin{aligned} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} &= \sqrt{e^{2\theta} (\cos \theta - \sin \theta)^2 + e^{2\theta} (\sin \theta + \cos \theta)^2} \\ &= \sqrt{2} e^\theta \end{aligned}$$

よて曲線の長さLは

$$\begin{aligned} L &= \sqrt{2} \int_0^\pi e^\theta d\theta \\ &= \sqrt{2} [e^\theta]_0^\pi \\ &= \sqrt{2} (e^\pi - 1) \end{aligned}$$

$$(4) y = \log(1-x^2) \quad (0 \leq x \leq \frac{1}{2})$$

$$\frac{dy}{dx} = -\frac{2x}{1-x^2}$$

$$\begin{aligned} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} &= \sqrt{1 + \frac{4x^2}{(1-x^2)^2}} \\ &= \sqrt{\frac{(1-x^2)^2 + 4x^2}{(1-x^2)^2}} = \frac{2 - (1-x^2)}{1-x^2} \\ &= \sqrt{\frac{(1+x^2)^2}{(1-x^2)^2}} = \frac{1+x^2}{1-x^2} = \frac{1}{1+x} + \frac{1}{1-x} - 1 \end{aligned}$$

よて曲線の長さLは

$$\begin{aligned} L &= \int_0^{\frac{1}{2}} \left(\frac{1}{1+x} + \frac{1}{1-x} - 1 \right) dx \\ &= \left[\log \frac{1+x}{1-x} - x \right]_0^{\frac{1}{2}} \\ &= \log 3 - \frac{1}{2} \end{aligned}$$

$$(5) y = \log(\cos x) \quad (0 \leq x \leq \frac{\pi}{3})$$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} = -\tan x$$

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \sqrt{1 + \tan^2 x} = \sqrt{\frac{1}{\cos^2 x}} = \frac{1}{\cos x}$$

よって曲線の長さLは

$$L = \int_0^{\frac{\pi}{3}} \frac{dx}{\cos x}$$

$$= \int_0^{\frac{\pi}{3}} \frac{\cos x}{1 - \sin^2 x} dx = (*)$$

$$\sin x = t \text{ とおくと } \frac{dt}{dx} = \cos x$$

$$x \mid 0 \rightarrow \frac{\pi}{3}$$

$$t \mid 0 \rightarrow \frac{\sqrt{3}}{2}$$

$$(*) = \int_0^{\frac{\sqrt{3}}{2}} \frac{dt}{1-t^2}$$

$$= \frac{1}{2} \int_0^{\frac{\sqrt{3}}{2}} \left(\frac{1}{1+t} + \frac{1}{1-t} \right) dt$$

$$= \frac{1}{2} [\log|1+t| - \log|1-t|]_0^{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2} \left[\log \frac{1+t}{1-t} \right]_0^{\frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2} \log \frac{1 + \frac{\sqrt{3}}{2}}{1 - \frac{\sqrt{3}}{2}}$$

$$= \frac{1}{2} \log \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$$

分母分子に
 $2 + \sqrt{3}$ を
 かける

$$= \frac{1}{2} \log (2 + \sqrt{3})^2$$

$$= \underline{\underline{\log(2 + \sqrt{3})}}$$