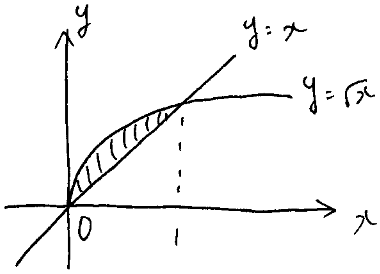
1. (1) $y=\sqrt{x}$, $y=x$




求める面積は図の斜線部分

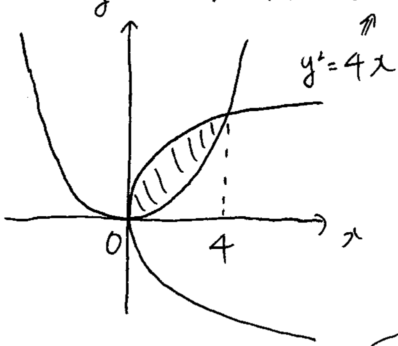
よって

$$S=\int_0^1(\sqrt{x}-x)\,dx$$

$$=\left[\frac{2}{3}x^{\frac{3}{2}}-\frac{1}{2}x^2\right]_0^1=\underline{\frac{1}{6}}$$

(2) $y^2=4x$, $x^2=4y$

$x^2=4y$　$y=2\sqrt{x}$ (※$y\geq 0$)



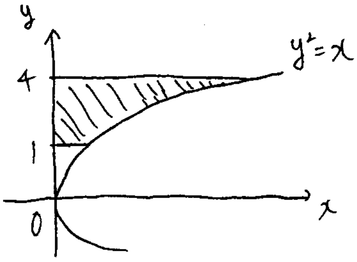

求める面積は図の斜線部分

よって

$$S=\int_0^4\left(2\sqrt{x}-\frac{1}{4}x^2\right)dx$$

$$=\left[\frac{4}{3}x^{\frac{3}{2}}-\frac{1}{12}x^3\right]_0^4=\underline{\frac{16}{3}}$$

(3) $y^2=x$, $y=1$, $y=4$, $y$軸

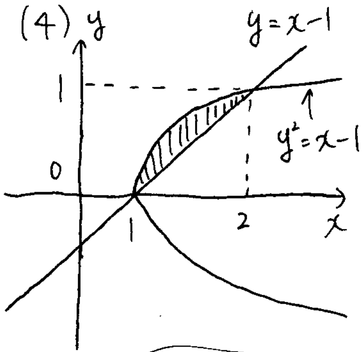



求める面積は図の斜線部分

よって

$$S=\int_1^4 y^2\,dy=\left[\frac{1}{3}y^3\right]_1^4=\underline{21}$$

(4)



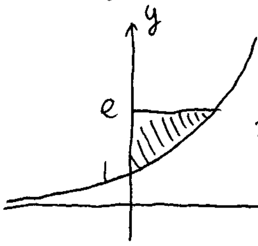

求める面積は図の斜線部分

よって

$$S=\int_0^1\{(y+1)-(y^2+1)\}\,dy$$

$$=\int_0^1(-y^2+y)\,dy=\underline{\frac{1}{6}}$$

(5) $y=e^x$, $y=e$, $y$軸




求める面積は図の斜線部分

よって

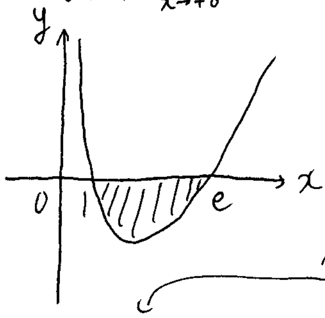
$$S=\int_1^e \log y\,dy$$

$$=\Big[y\log y-y\Big]_1^e=\underline{1}$$

(6) $y=(x-e)\log x$, $y=0$

$y=0$となる$x$は$x=1, e$

また、$\displaystyle\lim_{x\to+0}(x-e)\log x=\infty$




求める面積は図の斜線部分

よって

$$S=-\int_1^e(x-e)\log x\,dx$$

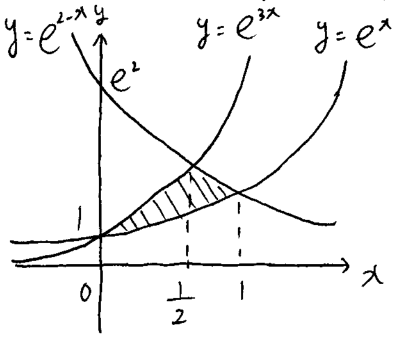
$$=-\int_1^e\left(\frac{1}{2}x^2-ex\right)'\log x\,dx$$

$$=-\left[\left(\frac{1}{2}x^2-ex\right)\log x\right]_1^e+\int\left(\frac{1}{2}x^{\cancel{2}}-e\cancel{x}\right)\cdot\frac{1}{\cancel{x}}\,dx$$

$$=\left[-\left(\frac{1}{2}x^2-ex\right)\log x+\frac{1}{4}x^2-ex\right]_1^e$$

$$=\underline{-\frac{1}{4}e^2+e-\frac{1}{4}}$$

(7) $y=e^x$. $y=e^{3x}$. $y=e^{2-x}$




求める面積は図の斜線部分

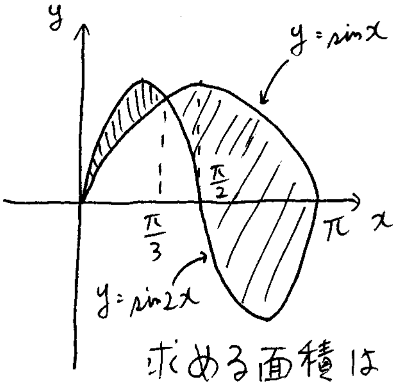
よって

$$S=\int_0^{\frac{1}{2}}(e^{3x}-e^x)dx+\int_{\frac{1}{2}}^{1}(e^{2-x}-e^x)dx$$

$$=\left[\frac{1}{3}e^{3x}-e^x\right]_0^{\frac{1}{2}}+\left[-e^{2-x}-e^x\right]_{\frac{1}{2}}^{1}$$

$$=\underline{\frac{4}{3}e^{\frac{3}{2}}-2e+\frac{2}{3}}$$

(8) $y=\sin x$. $y=\sin 2x$ $(0\leqq x\leqq\pi)$




交点は

$\sin x=\sin 2x$

$\sin x=2\sin x\cos x$

$\cos x=\frac{1}{2}$

$x=\frac{\pi}{3}$

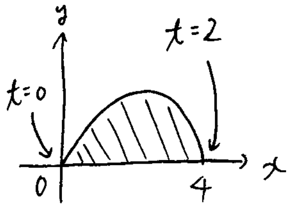
求める面積は図の斜線部分

よって, $$S=\int_0^{\frac{\pi}{3}}(\sin 2x-\sin x)dx+\int_{\frac{\pi}{3}}^{\pi}(\sin x-\sin 2x)dx$$

$$=\left[-\frac{1}{2}\cos 2x+\cos x\right]_0^{\frac{\pi}{3}}+\left[-\cos x+\frac{1}{2}\cos 2x\right]_{\frac{\pi}{3}}^{\pi}$$

$$=\underline{\frac{5}{2}}$$

2. (1) $x=2t$. $y=2t-t^2$ $(0\leqq t\leqq 2)$




求める面積は図の斜線部分

よって

$$S=\int_0^4 y\,dx$$

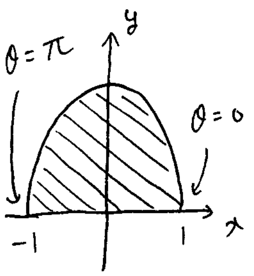
$$=\int_0^2 y\frac{dx}{dt}dt$$

| $x=2t$ | |
|---|---|
| $\frac{dx}{dt}=2$ | |
| $dx=2dt$ | |
| $x$ | $0\to 4$ |
| $t$ | $0\to 2$ |

$$=\int_0^2(2t-t^2)\cdot 2\,dt$$

$$=2\left[t^2-\frac{1}{3}t^3\right]_0^2=\underline{\frac{8}{3}}$$

(2) $x=\cos\theta$. $y=2\sin\theta$ $(0\leqq\theta\leqq\pi)$




求める面積は図の斜線部分

よって

$$S=\int_{-1}^{1}y\,dx$$

$$=\int_\pi^0 y\frac{dx}{d\theta}d\theta$$

| $x=\cos\theta$ | |
|---|---|
| $\frac{dx}{d\theta}=-\sin\theta$ | |
| $x$ | $-1\to 1$ |
| $\theta$ | $\pi\to 0$ |

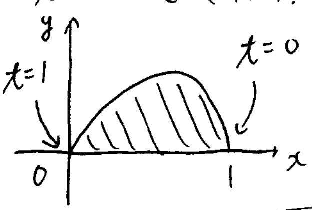
$$=\int_\pi^0 2\sin\theta\cdot(-\sin\theta)d\theta=2\int_0^\pi\sin^2\theta\,d\theta$$

$$=2\int_0^\pi\frac{1-\cos 2\theta}{2}d\theta=\underline{\pi}$$

(3) $x=1-t^4$. $y=t-t^3$ $(0\leqq t\leqq 1)$

$0\leqq t\leqq 1$ では $x\geqq 0$. $y\geqq 0$.

$t=0$ のとき $(1.0)$. $t=1$ のとき $(0.0)$




$$S=\int_0^1 y\,dx$$

$$=\int_1^0 y\frac{dx}{dt}dt$$

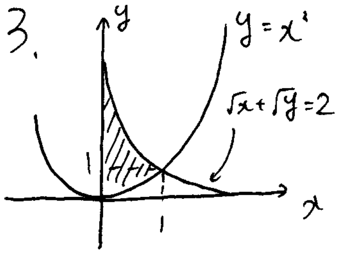
| $x=1-t^4$ | |
|---|---|
| $\frac{dx}{dt}=-4t^3$ | |
| $x$ | $0\to 1$ |
| $t$ | $1\to 0$ |

$$=\int_1^0(t-t^3)\cdot(-4t^3)\,dt=4\int_0^1 t^3(t-t^3)dt$$

$$=4\int_0^1(t^4-t^6)\,dt$$

$$=4\left[\frac{1}{5}t^5-\frac{1}{7}t^7\right]_0^1=\underline{\frac{8}{35}}$$

3.




求める面積は図の斜線部分

よって

$$S=\int_0^1\left\{(2-\sqrt{x})^2-x^2\right\}dx$$

$$=\int_0^1(-x^2+x-4\sqrt{x}+4)dx$$

$$=\left[-\frac{1}{3}x^3+\frac{1}{2}x^2-\frac{8}{3}x^{\frac{3}{2}}+4x\right]_0^1$$

$$=\underline{\frac{3}{2}}$$