1.

(1) $F(x) = \int_2^x (t+1)\log t\,dt$

$$F'(x) = \frac{d}{dx}\int_2^x (t+1)\log t\,dt$$

$$= \underline{(x+1)\log x}$$

(2) $F(x) = \int_x^{x^2} \underbrace{e^t \sin t}_{\text{原始関数を } f(x) \text{ とおくと}}\,dt$

$$= \left[f(t)\right]_x^{x^2} = f(x^2) - f(x)$$

$$F'(x) = 2x f'(x^2) - f'(x)$$

$$= \underline{2x e^{x^2}\sin x^2 - e^x \sin x}$$

(3) $F(x) = \int_1^x (t-x)\log t\,dt$

$$= \int_1^x t\log t\,dt - x\int_1^x \log t\,dt$$

$$F'(x) = x\log x - \left(\int_1^x \log t\,dt + x\log x\right)$$

$$= -\left[t\log t - t\right]_1^x$$

$$= \underline{-x\log x + x - 1}$$

(4) $F(x) = \int_x^{2x^2} (x+t)\sin t\,dt$

$$= x\int_x^{2x^2} \sin t\,dt + \int_x^{2x^2} t\sin t\,dt$$

$$F'(x) = \int_x^{2x^2} \sin t\,dt + x(4x\sin 2x^2 - \sin x) + 2x^2\sin 2x^2\cdot 4x - x\sin x$$

$$= \left[-\cos t\right]_x^{2x^2} + 4x^2\sin 2x^2 - x\sin x + 8x^3\sin 2x^2 - x\sin x$$

$$= \underline{-\cos 2x^2 + \cos x + 4x^2\sin 2x^2(2x+1) - 2x\sin x}$$

2.

(1) $f(x) = x + \underbrace{\int_0^2 f(t)e^t\,dt}_{a\text{とおく}}$

$$= x + a$$

$$a = \int_0^2 (t+a)e^t\,dt$$

$$= \left[(t+a)e^t\right]_0^2 - \int_0^2 e^t\,dt$$

$$= (2+a)e^2 - a - (e^2 - 1)$$

$$a = 2e^2 + ae^2 - a - e^2 + 1$$

$$2a - ae^2 = e^2 + 1$$

$$a(2 - e^2) = e^2 + 1$$

$$a = -\frac{e^2+1}{e^2-2}$$

よって $\underline{f(x) = x - \frac{e^2+1}{e^2-2}}$

(2) $f(x) = \log x - x\underbrace{\int_1^e \frac{f(t)+1}{t}\,dt}_{a\text{とおく}}$

$$= \log x - ax$$

$$a = \int_1^e \frac{\log t - at + 1}{t}\,dt$$

$$= \int_1^e \left(\frac{\log t}{t} - a + \frac{1}{t}\right)dt$$

$$= \left[\frac{1}{2}(\log t)^2 - at + \log t\right]_1^e$$

$$a = \frac{1}{2} - ae + 1 - (-a)$$

$$ae = \frac{3}{2} \qquad a = \frac{3}{2e}$$

よって

$$\underline{f(x) = \log x - \frac{3}{2e}x}$$

3.

(1) $\displaystyle \lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}e^{\frac{2k}{n}}$

$\displaystyle =\int_0^1 e^{2x}dx$

$\displaystyle =\left[\frac{1}{2}e^{2x}\right]_0^1=\underline{\frac{1}{2}(e^2-1)}$

(2) $\displaystyle \lim_{n\to\infty}\frac{1}{n}\left(\sin\frac{\pi}{2n}+\sin\frac{2\pi}{2n}+\cdots+\sin\frac{n\pi}{2n}\right)$

$\displaystyle =\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sin\frac{k\pi}{2n}$

$\displaystyle =\int_0^1\sin\frac{\pi}{2}x\,dx$

$\displaystyle =\left[-\frac{2}{\pi}\cos\frac{\pi}{2}x\right]_0^1=\underline{\frac{2}{\pi}}$

(3) $\displaystyle \lim_{n\to\infty}\frac{1}{n}\left\{\left(\frac{n}{n+1}\right)^2+\left(\frac{n}{n+2}\right)^2+\cdots+\left(\frac{n}{n+n}\right)^2\right\}$

$\displaystyle =\lim_{n\to\infty}\frac{1}{n}\left\{\frac{1}{(1+\frac{1}{n})^2}+\frac{1}{(1+\frac{2}{n})^2}+\cdots+\frac{1}{(1+\frac{n}{n})^2}\right\}$

$\displaystyle =\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{(1+\frac{k}{n})^2}$

$\displaystyle =\int_0^1\frac{dx}{(1+x)^2}$

$\displaystyle =\left[-\frac{1}{1+x}\right]_0^1=\underline{\frac{1}{2}}$

(4) $\displaystyle \lim_{n\to\infty}\frac{1}{n^2}\left\{(\sqrt{1}+\sqrt{n})^2+(\sqrt{2}+\sqrt{n})^2+\cdots+(\sqrt{n}+\sqrt{n})^2\right\}$

$\displaystyle =\lim_{n\to\infty}\frac{1}{n}\left\{\frac{(\sqrt{1}+\sqrt{n})^2}{n}+\frac{(\sqrt{2}+\sqrt{n})^2}{n}+\cdots+\frac{(\sqrt{n}+\sqrt{n})^2}{n}\right\}$

$\displaystyle =\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\left(\sqrt{\frac{k}{n}}+1\right)^2 \longleftarrow \frac{(\sqrt{k}+\sqrt{n})^2}{n}=\left(\frac{\sqrt{k}+\sqrt{n}}{\sqrt{n}}\right)^2=\left(\sqrt{\frac{k}{n}}+1\right)^2$

$\displaystyle =\int_0^1(\sqrt{x}+1)^2dx$

$\displaystyle =\int_0^1(x+2\sqrt{x}+1)dx$

$\displaystyle =\left[\frac{1}{2}x^2+\frac{4}{3}x\sqrt{x}+x\right]_0^1$

$\displaystyle =\underline{\frac{17}{6}}$