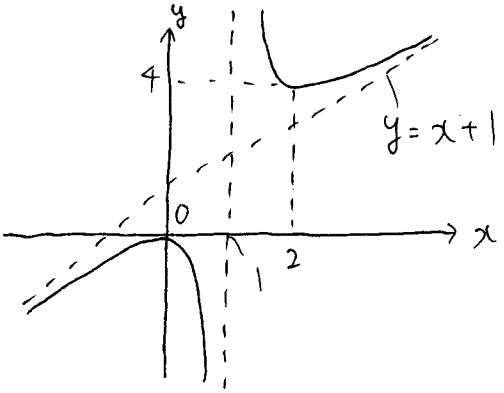
1. (1) $y=\dfrac{x^2}{x-1}=x+1+\dfrac{1}{x-1}$

$y'=1-\dfrac{1}{(x-1)^2}=\dfrac{x(x-2)}{(x-1)^2}$

x	$\cdots$	0	$\cdots$	1	$\cdots$	2	$\cdots$
y'	$+$	0	$-$	/	$-$	0	$+$
y	↗	0	↘	/	↘	4	↗

漸近線は $x=1$、$y=x+1$



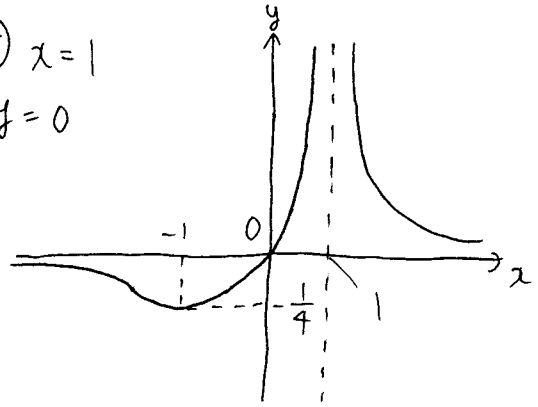
(2) $y=\dfrac{x}{(x-1)^2}$ $\lim_{x\to+\infty} y=0$

$y'=-\dfrac{x+1}{(x-1)^3}$ $\lim_{x\to-\infty} y=0$

x	$\cdots$	-1	$\cdots$	1	$\cdots$
y'	$-$	0	$+$	/	$-$
y	↘	$-\frac{1}{4}$	↗	/	↘

$\lim_{x\to1+0} y=\infty$

$\lim_{x\to1-0} y=\infty$



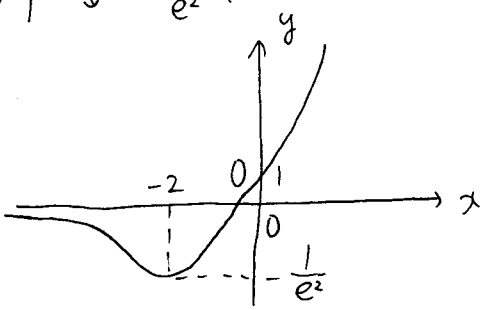
(3) $y=(x+1)e^x$

$y'=(x+2)e^x$

x	$\cdots$	-2	$\cdots$
y'	$-$	0	$+$
y	↘	$-\frac{1}{e^2}$	↗

$\lim_{x\to-\infty} y=0$

$\lim_{x\to+\infty} y=\infty$



(4) $y=|x-1|\sqrt{x+3}$

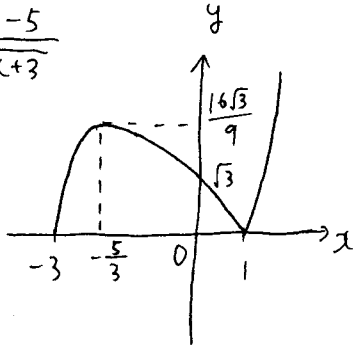
$x\geqq1$ のとき $y=(x-1)\sqrt{x+3}$ $x=-\frac{5}{3}$ は

$y'=\dfrac{3x+5}{2\sqrt{x+3}}$ $x\geqq1$ を満たさない

$x\leqq1$ のとき $y=-(x-1)\sqrt{x+3}$

$y'=\dfrac{-3x-5}{2\sqrt{x+3}}$

x	-3	$\cdots$	$-\frac{5}{3}$	$\cdots$	1	$\cdots$
y'	/	$+$	0	$-$	/	$+$
y	0	↗	$\frac{16\sqrt{3}}{9}$	↘	0	↗



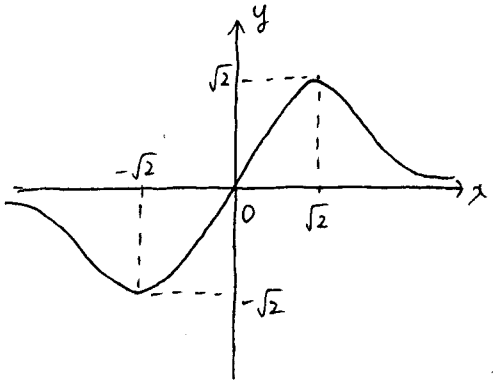
(5) $y=\dfrac{4x}{x^2+2}$

$y'=-\dfrac{4(x^2-2)}{(x^2+2)^2}$

x	$\cdots$	$-\sqrt{2}$	$\cdots$	$\sqrt{2}$	$\cdots$
y'	$-$	0	$+$	0	$-$
y	↘	$-\sqrt{2}$	↗	$\sqrt{2}$	↘

$\lim_{x\to+\infty} y=0$

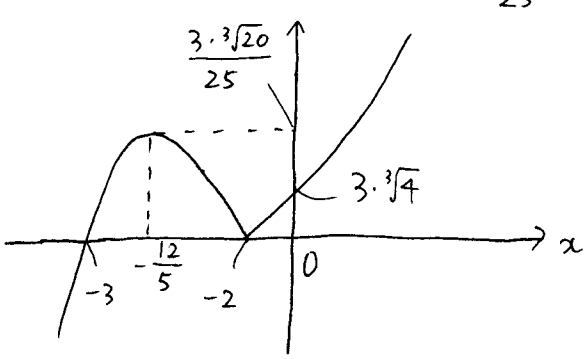
$\lim_{x\to-\infty} y=0$



(6) $y=(x+3)\sqrt[3]{(x+2)^2}$

$y'=\dfrac{5x+12}{3\cdot\sqrt[3]{x+2}}$

x	$\cdots$	$-\frac{12}{5}$	$\cdots$	-2	$\cdots$
y'	$+$	0	$-$	/	$+$
y	↗	$\frac{3\cdot\sqrt[3]{20}}{25}$	↘	0	↗



2. $y = x + \dfrac{a}{x-1}$

$y' = 1 - \dfrac{a}{(x-1)^2}$

$= \dfrac{(x-1)^2 - a}{(x-1)^2}$

$y' = 0$ となる x は $x = 1 \pm \sqrt{a}$

x	$\cdots$	$1-\sqrt{a}$	$\cdots$	$\cdots$	$1+\sqrt{a}$	$\cdots$
y'	+	0	−	−	0	+
y	↗	$1-2\sqrt{a}$	↘	↘	$1+2\sqrt{a}$	↗

ここが -1 になる

よって $1-2\sqrt{a} = -1$

$\sqrt{a} = 1$ $\quad a = 1$

3. $f(x) = \dfrac{ax^2+bx+1}{x^2+1}$

$f'(x) = \dfrac{(2ax+b)(x^2+1) - (ax^2+bx+1)\cdot 2x}{(x^2+1)^2}$

$= \dfrac{(2ax^3+bx^2+2ax+b) - (2ax^3+2bx^2+2x)}{(x^2+1)^2}$

$= \dfrac{-bx^2+(2a-2)x+b}{(x^2+1)^2}$

$f'(2) = 0$ より $\quad 4a - 3b = 4 \cdots ①$

$f(2) = -1$ より $\quad 2a + b = -3 \cdots ②$

①, ② より $\quad a = -\dfrac{1}{2}$, $b = -2$

これより

$f'(x) = \dfrac{2x^2-3x-2}{(x^2+1)^2}$

$= \dfrac{(2x+1)(x-2)}{(x^2+1)^2}$

x	$\cdots$	$-\frac{1}{2}$	$\cdots$	2	$\cdots$
y'	+	0	−	0	+
y	↗	極大	↘	-1	↗

よって $f\left(-\dfrac{1}{2}\right) = \dfrac{-\frac{1}{2}\left(-\frac{1}{2}\right)^2 + 1 + 1}{\frac{1}{4}+1}$

$= \dfrac{-\frac{1}{8}+2}{\frac{5}{4}} = \dfrac{\frac{15}{8}}{\frac{5}{4}}$

$= \dfrac{3}{2}$

極大値 $x = -\dfrac{1}{2}$ で $\dfrac{3}{2}$