(1) $\int_0^1 x(x-1)^4 dx$

$= \int_0^1 x\left(\frac{1}{5}(x-1)^5\right)' dx$

$= \left[\frac{1}{5}x(x-1)^5\right]_0^1 - \int_0^1 \frac{1}{5}(x-1)^5 dx$

$= -\frac{1}{30}\left[(x-1)^6\right]_0^1 = \underline{\frac{1}{30}}$

(2) $\int_0^{\frac{\pi}{2}} (x+2)\cos x\, dx$

$= \int_0^{\frac{\pi}{2}} (x+2)(\sin x)' dx$

$= \left[(x+2)\sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x\, dx$

$= \left[(x+2)\sin x + \cos x\right]_0^{\frac{\pi}{2}} = \underline{\frac{\pi}{2} + 1}$

(3) $\int_1^2 x^4 \log x\, dx$

$= \int_1^2 \left(\frac{1}{5}x^5\right)' \log x\, dx$

$= \left[\frac{1}{5}x^5 \log x\right]_1^2 - \frac{1}{5}\int_1^2 x^5 \cdot \frac{1}{x} dx$

$= \left[\frac{1}{5}x^5 \log x - \frac{1}{25}x^5\right]_1^2$

$= \left(\frac{32}{5}\log 2 - \frac{32}{25}\right) - \left(0 - \frac{1}{25}\right)$

$= \underline{\frac{32}{5}\log 2 - \frac{31}{25}}$

(4) $\int_1^e \frac{\log x}{x^2} dx$

$= \int_1^e \log x \cdot \left(-\frac{1}{x}\right)' dx$

$= \left[-\frac{\log x}{x}\right]_1^e + \int_1^e \frac{dx}{x^2}$

$= \left[-\frac{\log x}{x} - \frac{1}{x}\right]_1^e$

$= \left(-\frac{1}{e} - \frac{1}{e}\right) - (0 - 1)$

$= \underline{-\frac{2}{e} + 1}$

(5) $\int_0^{\frac{\pi}{3}} \frac{x}{\cos^2 x} dx$

$= \int_0^{\frac{\pi}{3}} x(\tan x)' dx$

$= \left[x\tan x\right]_0^{\frac{\pi}{3}} - \int \tan x\, dx$

$= \left[x\tan x + \log|\cos x|\right]_0^{\frac{\pi}{3}}$

$= \left(\frac{\pi}{3}\cdot\sqrt{3} + \log\frac{1}{2}\right) - (0 + 0)$

$= \underline{\frac{\sqrt{3}}{3}\pi - \log 2}$

(6) $\int_0^1 x^2 e^{2x} dx$

$= \int_0^1 x^2 \left(\frac{1}{2}e^{2x}\right)' dx$

$= \left[\frac{1}{2}x^2 e^{2x}\right]_0^1 - \int_0^1 x e^{2x} dx$

$=$ 〃 $- \int_0^1 x\left(\frac{1}{2}e^{2x}\right)' dx$

$=$ 〃 $- \left\{\left[\frac{1}{2}x e^{2x}\right]_0^1 - \frac{1}{2}\int_0^1 e^{2x} dx\right\}$

$= \left[\frac{1}{2}x^2 e^{2x} - \frac{1}{2}x e^{2x} + \frac{1}{4}e^{2x}\right]_0^1$

$= \left(\frac{1}{2}e^2 - \frac{1}{2}e^2 + \frac{1}{4}e^2\right) - \left(\frac{1}{4}\right)$

$= \frac{1}{4}e^2 - \frac{1}{4} = \underline{\frac{1}{4}(e^2 - 1)}$