(1) $\int_{-1}^{0}(3x+2)^5\,dx$

$= \left[\frac{1}{6}\cdot\frac{1}{3}(3x+2)^6\right]_{-1}^{0}$

$= \frac{1}{18}(64-1) = \frac{63}{18} = \underline{\frac{7}{2}}$

(2) $\int_{-1}^{1}\frac{dx}{\sqrt{x+2}}$

$x+2=t$ とおくと $\frac{dt}{dx}=1$

$\begin{array}{c|c} x & -1 \to 1 \\ \hline t & 1 \to 3 \end{array}$ $dx=dt$

$= \int_{1}^{3}\frac{dt}{\sqrt{t}}$

$= \left[2\sqrt{t}\right]_{1}^{3} = \underline{2(\sqrt{3}-1)}$

(3) $\int_{0}^{3}x\sqrt{x+1}\,dx$

$x+1=t$ とおくと

$x=t-1$

$\frac{dt}{dx}=1$ $dx=dt$

$\begin{array}{c|c} x & 0 \to 3 \\ \hline t & 1 \to 4 \end{array}$

$= \int_{1}^{4}(t-1)\sqrt{t}\,dt$

$= \int_{1}^{4}(t\sqrt{t}-\sqrt{t})\,dt$

$= \left[\frac{2}{5}t^2\sqrt{t}-\frac{2}{3}t\sqrt{t}\right]_{1}^{4}$

$= \left(\frac{64}{5}-\frac{16}{3}\right)-\left(\frac{2}{5}-\frac{2}{3}\right) = \underline{\frac{116}{15}}$

(4) $\int_{-2}^{2\sqrt{3}}\frac{dx}{\sqrt{16-x^2}}$

$x=4\sin\theta$ とおくと

$\frac{dx}{d\theta}=4\cos\theta$

$dx=4\cos\theta\,d\theta$

$\begin{array}{c|c} x & -2 \to 2\sqrt{3} \\ \hline \theta & -\frac{\pi}{6} \to \frac{\pi}{3} \end{array}$

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{4\cos\theta}{\sqrt{16-16\sin^2\theta}}\,d\theta$

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{4\cos\theta}{4\cos\theta}\,d\theta$

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{3}}d\theta = \frac{\pi}{3}-\left(-\frac{\pi}{6}\right) = \underline{\frac{\pi}{2}}$

(5) $\int_{-1}^{1}\frac{dx}{x^2+1}$

$x=\tan\theta$ とおくと

$\frac{dx}{d\theta}=\frac{1}{\cos^2\theta}$ $dx=\frac{d\theta}{\cos^2\theta}$

$\begin{array}{c|c} x & -1 \to 1 \\ \hline \theta & -\frac{\pi}{4} \to \frac{\pi}{4} \end{array}$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{\tan^2\theta+1}\cdot\frac{d\theta}{\cos^2\theta}$

$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}d\theta = \frac{\pi}{4}-\left(-\frac{\pi}{4}\right) = \underline{\frac{\pi}{2}}$

(6) $\int_{0}^{2\sqrt{3}}\frac{dx}{3x^2+12}$

$x=2\tan\theta$ とおくと

$\frac{dx}{d\theta}=\frac{2}{\cos^2\theta}$

$dx=\frac{2}{\cos^2\theta}d\theta$

$\begin{array}{c|c} x & 0 \to 2\sqrt{3} \\ \hline \theta & 0 \to \frac{\pi}{3} \end{array}$

$= \frac{1}{3}\int_{0}^{2\sqrt{3}}\frac{dx}{x^2+4}$

$= \frac{1}{3}\int_{0}^{\frac{\pi}{3}}\frac{1}{4\tan^2\theta+4}\cdot\frac{2}{\cos^2\theta}d\theta$

$= \frac{1}{6}\int_{0}^{\frac{\pi}{3}}d\theta = \frac{1}{6}\cdot\frac{\pi}{3} = \underline{\frac{\pi}{18}}$

(7) $\int_{0}^{\sqrt{3}}\frac{x^2}{x^2+9}\,dx$

$x=3\tan\theta$ とおくと

$\frac{dx}{d\theta}=\frac{3}{\cos^2\theta}$

$dx=\frac{3}{\cos^2\theta}d\theta$

$\begin{array}{c|c} x & 0 \to \sqrt{3} \\ \hline \theta & 0 \to \frac{\pi}{6} \end{array}$

$= \int_{0}^{\sqrt{3}}\left(1-\frac{9}{x^2+9}\right)dx$

$= \sqrt{3}-9\int_{0}^{\sqrt{3}}\frac{dx}{x^2+9}$

$= \sqrt{3}-9\int_{0}^{\frac{\pi}{6}}\frac{1}{9\tan^2\theta+9}\cdot\frac{3}{\cos^2\theta}d\theta$

$= \sqrt{3}-3\int_{0}^{\frac{\pi}{6}}d\theta$

$= \sqrt{3}-3\cdot\frac{\pi}{6} = \underline{\sqrt{3}-\frac{\pi}{2}}$

(8) $\int_{1}^{2}\frac{x^2-2x}{x^3-3x^2+1}\,dx$

$= \int_{1}^{2}\frac{\frac{1}{3}(x^3-3x^2+1)'}{x^3-3x^2+1}\,dx$

$= \frac{1}{3}\left[\log|x^3-3x^2+1|\right]_{1}^{2}$

$= \underline{\frac{1}{3}\log 3}$

(9) $\displaystyle\int_e^{e^2} \frac{dx}{x\log x}$

$\log x = t$ とおくと

$\dfrac{dt}{dx} = \dfrac{1}{x}$

$\dfrac{dx}{x} = dt$

$\begin{array}{c|c} x & e \to e^2 \\ \hline t & 1 \to 2 \end{array}$

$\displaystyle = \int_1^2 \frac{dt}{t}$

$\displaystyle = [\log t]_1^2 = \underline{\log 2}$

(10) $\displaystyle\int_0^1 x^2 e^{x^3} dx$

$x^3 = t$ とおくと

$\dfrac{dt}{dx} = 3x^2 \Rightarrow x^2 dx = \dfrac{1}{3}dt$

$\begin{array}{c|c} x & 0 \to 1 \\ \hline t & 0 \to 1 \end{array}$

$\displaystyle = \frac{1}{3}\int_0^1 e^t dt$

$\displaystyle = \frac{1}{3}[e^t]_0^1 = \underline{\frac{1}{3}(e-1)}$

(11) $\displaystyle\int_0^4 \frac{x^2}{\sqrt{x+1}} dx$

$x+1 = t$ とおくと

$x = t-1$

$\dfrac{dx}{dt} = 1 \quad dx = dt$

$\begin{array}{c|c} x & 0 \to 4 \\ \hline t & 1 \to 5 \end{array}$

$\displaystyle = \int_1^5 \frac{(t-1)^2}{\sqrt{t}} dt$

$\displaystyle = \int_1^5 \left(\sqrt{t^3} - 2\sqrt{t} + \frac{1}{\sqrt{t}}\right) dt$

$\displaystyle = \left[\frac{2}{5}\sqrt{t^5} - \frac{4}{3}\sqrt{t^3} + 2\sqrt{t}\right]_1^5$

$\displaystyle = \left(10\sqrt{5} - \frac{20}{3}\sqrt{5} + 2\sqrt{5}\right) - \left(\frac{2}{5} - \frac{4}{3} + 2\right)$

$\displaystyle = \underline{\frac{16}{3}\sqrt{5} - \frac{16}{15}}$

(12) $\displaystyle\int_1^2 \frac{dx}{e^x - 1}$

$e^x - 1 = t$ とおくと

$\dfrac{dt}{dx} = e^x = t+1$

$\begin{array}{c|c} x & 1 \to 2 \\ \hline t & e-1 \to e^2-1 \end{array}$

$dx = \dfrac{dt}{t+1}$

$\displaystyle = \int_{e-1}^{e^2-1} \frac{1}{t}\cdot\frac{dt}{t+1}$

$\displaystyle = \int_{e-1}^{e^2-1} \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$

$\displaystyle = \left[\log\frac{t}{t+1}\right]_{e-1}^{e^2-1}$

$\displaystyle = \log\frac{e^2-1}{e^2} - \log\frac{e-1}{e}$

$\displaystyle = \log(e^2-1) - 2 - \log(e-1) + 1$

$\displaystyle = \log\frac{e^2-1}{e-1} - 1 = \underline{\log(e+1) - 1}$

(13) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin^3 x}{\cos^2 x} dx$

$\cos x = t$ とおくと

$\dfrac{dt}{dx} = -\sin x$

$\sin x\, dx = -dt$

$\begin{array}{c|c} x & 0 \to \frac{\pi}{4} \\ \hline t & 1 \to \frac{1}{\sqrt{2}} \end{array}$

$\displaystyle = \int_0^{\frac{\pi}{4}} \frac{(1-\cos^2 x)\sin x}{\cos^2 x} dx$

$\displaystyle = \int_1^{\frac{1}{\sqrt{2}}} \frac{1-t^2}{t^2}(-1) dt$

$\displaystyle = \int_{\frac{1}{\sqrt{2}}}^1 \left(\frac{1}{t^2} - 1\right) dt$

$\displaystyle = \left[-\frac{1}{t} - t\right]_{\frac{1}{\sqrt{2}}}^1$

$\displaystyle = (-1-1) - \left(-\sqrt{2} - \frac{1}{\sqrt{2}}\right)$

$\displaystyle = \underline{-2 + \frac{3\sqrt{2}}{2}}$

(14) $\displaystyle\int_0^1 \sqrt{2x - x^2}\, dx$

$x - 1 = \sin\theta$ とおくと

$\dfrac{dx}{d\theta} = \cos\theta$

$dx = \cos\theta\, d\theta$

$\begin{array}{c|c} x & 0 \to 1 \\ \hline \theta & -\frac{\pi}{2} \to 0 \end{array}$

$\displaystyle = \int_0^1 \sqrt{1-(x-1)^2}\, dx$

$\displaystyle = \int_{-\frac{\pi}{2}}^0 \cos^2\theta\, d\theta$

$\displaystyle = \int_{-\frac{\pi}{2}}^0 \frac{1+\cos 2\theta}{2} d\theta$

$\displaystyle = \left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_{-\frac{\pi}{2}}^0$

$\displaystyle = \underline{\frac{\pi}{4}}$

(15) $\displaystyle\int_{-1}^{0} \frac{dx}{(x+1)^2+1}$

$x=\tan\theta-1$とおくと

$\dfrac{dx}{d\theta}=\dfrac{1}{\cos^2\theta}$

$dx=\dfrac{1}{\cos^2\theta}d\theta$

x	$-1 \to 0$
θ	$0 \to \frac{\pi}{4}$

$$=\int_{0}^{\frac{\pi}{4}} \frac{1}{\tan^2\theta+1}\cdot\frac{d\theta}{\cos^2\theta}$$

$$=\int_{0}^{\frac{\pi}{4}} d\theta$$

$$=\underline{\frac{\pi}{4}}$$

(16) $\displaystyle\int_{1}^{\sqrt{3}} \frac{2x+1}{x^2+1}dx$

$$=\underbrace{\int_{1}^{\sqrt{3}} \frac{2x}{x^2+1}dx}_{(\mathrm{i})}+\underbrace{\int_{1}^{\sqrt{3}} \frac{dx}{x^2+1}}_{(\mathrm{ii})}$$

$$(\mathrm{i})=\int_{1}^{\sqrt{3}} \frac{(x^2+1)'}{x^2+1}dx$$

$$=\Big[\log(x^2+1)\Big]_{1}^{\sqrt{3}}$$

$$=\log 4-\log 2=\log 2$$

(ii) $x=\tan\theta$とおくと $\dfrac{dx}{d\theta}=\dfrac{1}{\cos^2\theta}$

x	$1 \to \sqrt{3}$
θ	$\frac{\pi}{4} \to \frac{\pi}{3}$

$dx=\dfrac{d\theta}{\cos^2\theta}$

$$(\mathrm{ii})=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{1}{\tan^2\theta+1}\cdot\frac{d\theta}{\cos^2\theta}$$

$$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} d\theta=\frac{\pi}{3}-\frac{\pi}{4}=\frac{\pi}{12}$$

よって，

$$\int_{1}^{\sqrt{3}} \frac{2x+1}{x^2+1}dx=\underline{\log 2+\frac{\pi}{12}}$$