(1) $\int \frac{x^2+3x+4}{x+2}\,dx$

$= \int \left(x+1+\frac{2}{x+2}\right)dx$

$= \frac{1}{2}x^2+x+2\log|x+2|+C$

(2) $\int \frac{x^3}{x-1}\,dx$

$= \int \left(x^2+x+1+\frac{1}{x-1}\right)dx$

$= \frac{1}{3}x^3+\frac{1}{2}x^2+x+\log|x-1|+C$

(3) $\int \frac{dx}{x^2-4}$

$= \int \frac{dx}{(x+2)(x-2)}$

$= \int \frac{1}{4}\left(\frac{1}{x-2}-\frac{1}{x+2}\right)dx$

$= \frac{1}{4}\log\left|\frac{x-2}{x+2}\right|+C$

(4) $\int \frac{x}{(x-1)(2x-1)}\,dx$

$= \int \left(\frac{1}{x-1}-\frac{1}{2x-1}\right)dx$

$= \log|x-1|-\frac{1}{2}\log|2x-1|+C$

(5) $\int (e^x+1)^2\,dx$

$= \int (e^{2x}+2e^x+1)\,dx$

$= \frac{1}{2}e^{2x}+2e^x+x+C$

(6) $\int \frac{\sin^2 x}{1-\cos x}\,dx$

$= \int (1+\cos x)\,dx$

$= x+\sin x+C$

(7) $\int \sin 2x \sin 4x\,dx$

$= -\frac{1}{2}\int (\cos 6x-\cos 2x)\,dx$

$= -\frac{1}{12}\sin 6x+\frac{1}{4}\sin 2x+C$

(8) $\int \cos^2\frac{x}{2}\,dx$

$= \int \frac{1+\cos x}{2}\,dx$

$= \frac{1}{2}x+\frac{1}{2}\sin x+C$

(9) $\int \left(\tan^2 x+\frac{1}{\tan^2 x}\right)dx$

$= \int \left(\frac{1}{\cos^2 x}-1+\frac{1}{\sin^2 x}-1\right)dx$

$= \int \left(\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}-2\right)dx$

$= \tan x-\frac{1}{\tan x}-2x+C$

(10) $\int \frac{x^2+x+1}{x^2+1}\,dx$

$= \int \left(1+\frac{x}{x^2+1}\right)dx$

$= \int \left(1+\frac{1}{2}\cdot\frac{(x^2+1)'}{x^2+1}\right)dx$

$= x+\frac{1}{2}\log(x^2+1)+C$

(11) $\int \frac{x^4}{x^2-1}\,dx$

$= \int \left(x^2+1+\frac{1}{x^2-1}\right)dx$

$= \int \left\{x^2+1+\frac{1}{2}\left(\frac{1}{x-1}-\frac{1}{x+1}\right)\right\}dx$

$= \frac{1}{3}x^3+x+\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+C$

(12) $\int \frac{x^3}{x^2-4}\,dx$

$= \int \left(x+\frac{4x}{x^2-4}\right)dx$

$= \int \left(x+\frac{2}{x+2}+\frac{2}{x-2}\right)dx$

$= \frac{1}{2}x^2+\log(x+2)^2(x-2)^2+C$

(13) $\int \frac{dx}{x(x^2-1)}$

$= \int \frac{dx}{x(x+1)(x-1)}$

$\frac{1}{x(x+1)(x-1)} = \frac{a}{x}+\frac{b}{x-1}+\frac{c}{x+1}$ となるa, b, cを求める

$= \int \frac{1}{2}\left(\frac{1}{x+1}+\frac{1}{x-1}-\frac{2}{x}\right)dx$

$= \frac{1}{2}\left(\log|x+1|+\log|x-1|-2\log|x|\right)+C$

$= \frac{1}{2}\log\frac{|x^2-1|}{x^2}+C$

(14) $\int \frac{dx}{x(x^2+1)}$

$= \int \left(\frac{1}{x}-\frac{x}{x^2+1}\right)dx$

$= \log|x| - \frac{1}{2}\log|x^2+1| + C$

$= \frac{1}{2}\log x^2 - \frac{1}{2}\log(x^2+1)+C$

$= \frac{1}{2}\log\frac{x^2}{x^2+1}+C$

(15) $\int \frac{x^2+1}{x^4-5x^2+4}\,dx$

$= \int \frac{x^2+1}{(x+1)(x-1)(x+2)(x-2)}\,dx$

$\frac{x^2+1}{(x+1)(x-1)(x+2)(x-2)} = \frac{a}{x+1}+\frac{b}{x-1}+\frac{c}{x+2}+\frac{d}{x-2}$

となるa, b, c, dを求める

$a=\frac{1}{3}$, $b=-\frac{1}{3}$, $c=-\frac{5}{12}$, $d=\frac{5}{12}$

$= \int \left\{\frac{1}{3}\left(\frac{1}{x+1}-\frac{1}{x-1}\right)-\frac{5}{12}\left(\frac{1}{x+2}-\frac{1}{x-2}\right)\right\}dx$

$= \frac{1}{3}\log\left|\frac{x+1}{x-1}\right|+\frac{5}{12}\log\left|\frac{x-2}{x+2}\right|+C$

(16) $\int \frac{x^4}{x^3-3x+2}\,dx$

$= \int \left(x+\frac{3x^2-2x}{x^3-3x+2}\right)dx$

$= \int \left(x+\frac{3x^2-2x}{(x+2)(x-1)^2}\right)dx$

$= \int \left(x+\frac{16}{9}\cdot\frac{1}{x+2}+\frac{11}{9}\cdot\frac{1}{x-1}+\frac{1}{3}\cdot\frac{1}{(x-1)^2}\right)dx$

$= \frac{1}{2}x^2+\frac{16}{9}\log|x+2|+\frac{11}{9}\log|x-1|-\frac{1}{3(x-1)}+C$

$\frac{3x^2-2x}{(x+2)(x-1)^2} = \frac{a}{x+2}+\frac{b}{x-1}+\frac{c}{(x-1)^2}$

となるa, b, cを求める

$a=\frac{16}{9}$, $b=\frac{11}{9}$, $c=\frac{1}{3}$

(17) $\displaystyle\int \frac{x}{\sqrt{3x+4}-2}\,dx = \int \frac{1}{t-2}\cdot\frac{t^2-4}{3}\cdot\frac{2}{3}t\,dt$

$\sqrt{3x+4}=t$ とおくと

$3x+4=t^2$

$x=\dfrac{t^2-4}{3}$

$\dfrac{dx}{dt}=\dfrac{2}{3}t$

$dx=\dfrac{2}{3}t\,dt$

$\displaystyle = \int \frac{1}{t-2}\cdot\frac{(t+2)(t-2)}{3}\cdot\frac{2}{3}t\,dt$

$\displaystyle = \frac{2}{9}\int t(t+2)\,dt = \frac{2}{9}\int (t^2+2t)\,dt$

$\displaystyle = \frac{2}{9}\left(\frac{1}{3}t^3+t^2\right)+C$

$\displaystyle = \frac{2}{27}(t^3+3t^2)+C$

$\displaystyle = \frac{2}{27}t^2(t+3)+C$

$\displaystyle = \frac{2}{27}(3x+4)(\sqrt{3x+4}+3)+C$

(18) $\displaystyle\int \frac{\sqrt{x}}{\sqrt[4]{x^3+1}}\,dx = \int \frac{\sqrt[3]{t^2}}{t+1}\cdot\frac{4}{3}\cdot\sqrt[3]{t}\,dt$

$\sqrt[4]{x^3}=t$ とおくと

$x^3=t^4$

$x=\sqrt[3]{t^4}$

$\dfrac{dx}{dt}=\dfrac{4}{3}\cdot\sqrt[3]{t}$

$dx=\dfrac{4}{3}\cdot\sqrt[3]{t}\,dt$

$\displaystyle = \frac{4}{3}\int \frac{t}{t+1}\,dt$

$\displaystyle = \frac{4}{3}\int \left(1-\frac{1}{t+1}\right)dt$

$\displaystyle = \frac{4}{3}\left(t-\log|t+1|\right)+C$

$\displaystyle = \frac{4}{3}\left(\sqrt[4]{x^3}-\log|\sqrt[4]{x^3}+1|\right)+C$

(19) $\displaystyle\int \frac{dx}{x\sqrt{x+1}} = \int \frac{1}{t(t^2-1)}\cdot 2t\,dt$

$\sqrt{x+1}=t$ とおくと

$x+1=t^2$

$x=t^2-1$

$\dfrac{dx}{dt}=2t$

$dx=2t\,dt$

$\displaystyle = \int \frac{2}{(t+1)(t-1)}\,dt$

$\displaystyle = \int \left(\frac{1}{t-1}-\frac{1}{t+1}\right)dt$

$= \log|t-1|-\log|t+1|+C$

$\displaystyle = \log\left|\frac{t-1}{t+1}\right|+C$

$\displaystyle = \log\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right|+C$

(20) $\int \log|x^2-1|\,dx = \int \log|(x+1)(x-1)|\,dx$

$= \int (\log|x+1| + \log|x-1|)\,dx$

$= (x+1)\log|x+1| - x + (x-1)\log|x-1| - x + C$

$= \underline{(x+1)\log|x+1| + (x-1)\log|x-1| - 2x + C}$

(21) $\int \frac{e^x}{e^x - e^{-x}}\,dx = \int \frac{1}{t-\frac{1}{t}}\,dt$

$e^x = t$ とおくと

$\frac{dt}{dx} = e^x$

$e^x dx = dt$

$= \int \frac{t}{t^2-1}\,dt = \int \frac{t}{(t+1)(t-1)}\,dt$

$= \int \frac{1}{2}\left(\frac{1}{t+1} + \frac{1}{t-1}\right)dt$

$= \frac{1}{2}\log|(t+1)(t-1)| + C$

$= \underline{\frac{1}{2}\log|e^{2x}-1| + C}$

(22) $\int \frac{\cos x}{\sin x(\sin x+1)}\,dx = \int \frac{dt}{t(t+1)}$

$\sin x = t$ とおくと

$\frac{dt}{dx} = \cos x$

$\cos x\,dx = dt$

$= \int \left(\frac{1}{t} - \frac{1}{t+1}\right)dt$

$= \log|t| - \log|t+1| + C$

$= \underline{\log\frac{|\sin x|}{\sin x+1} + C}$

(23) $\int \tan^4 x\,dx$

$= \int \tan^2 x\left(\frac{1}{\cos^2 x} - 1\right)dx$

$= \int \left(\frac{\tan^2 x}{\cos^2 x} - \tan^2 x\right)dx$

$= \int \left(\frac{\tan^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} + 1\right)dx$

$= \underline{\frac{1}{3}\tan^3 x - \tan x + x + C}$

$\tan x = t$ とおくと

$\frac{dt}{dx} = \frac{1}{\cos^2 x}$

$\frac{dx}{\cos^2 x} = dt$

(24) $\int \frac{1}{1-\sin x}\,dx$

$= \int \frac{1+\sin x}{\cos^2 x}\,dx$

$= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right)dx$

$= \underline{\tan x + \frac{1}{\cos x} + C}$

$\cos x = t$ とおくと

$\frac{dt}{dx} = -\sin x$

$\sin x\,dx = -dt$