

$$\begin{aligned}
 (1) \int x \sin 2x \, dx & \\
 &= \int x \left(-\frac{1}{2} \cos 2x\right)' \, dx \\
 &= -\frac{1}{2} x \cos 2x + \frac{1}{2} \int \cos 2x \, dx \\
 &= -\frac{1}{2} x \cos 2x + \frac{1}{4} \sin 2x + C
 \end{aligned}$$

$$\begin{aligned}
 (2) \int \frac{x}{\cos^2 x} \, dx & \\
 &= \int x (\tan x)' \, dx \\
 &= x \tan x - \int \tan x \, dx \\
 &= x \tan x + \log |\cos x| + C
 \end{aligned}$$

$$\begin{aligned}
 (3) \int x^3 \log x \, dx & \\
 &= \int \left(\frac{1}{4} x^4\right)' \log x \, dx \\
 &= \frac{1}{4} x^4 \log x - \frac{1}{4} \int x^4 \cdot \frac{1}{x} \, dx \\
 &= \frac{1}{4} x^4 \log x - \frac{1}{4} \int x^3 \, dx \\
 &= \frac{1}{4} x^4 \log x - \frac{1}{16} x^4 + C
 \end{aligned}$$

$$\begin{aligned}
 (4) \int (2x-1)e^x \, dx & \\
 &= \int (2x-1) \cdot (e^x)' \, dx \\
 &= (2x-1)e^x - \int 2e^x \, dx \\
 &= (2x-1)e^x - 2e^x + C \\
 &= (2x-3)e^x + C
 \end{aligned}$$

$$\begin{aligned}
 (5) \int \log(1-x) \, dx & \\
 &= \int (x-1)' \log(1-x) \, dx \\
 &= (x-1) \log(1-x) - \int (x-1) \cdot \frac{1}{1-x} \cdot (-1) \, dx \\
 &= (x-1) \log(1-x) - \int dx \\
 &= (x-1) \log(1-x) - x + C
 \end{aligned}$$

$$\begin{aligned}
 (6) \int x^2 \cos x \, dx & \\
 &= \int x^2 (\sin x)' \, dx \\
 &= x^2 \sin x - \int 2x \sin x \, dx \\
 &= x^2 \sin x - 2 \int x (-\cos x)' \, dx \\
 &= x^2 \sin x - 2 \left\{ -x \cos x + \int \cos x \, dx \right\} \\
 &= x^2 \sin x + 2x \cos x - 2 \sin x + C \\
 &= (x^2-2) \sin x + 2x \cos x + C
 \end{aligned}$$

$$\begin{aligned}
 (7) \int x \log(x^2-2) \, dx & \\
 &= \int \left(\frac{1}{2} x^2-1\right)' \log(x^2-2) \, dx \\
 &= \left(\frac{1}{2} x^2-1\right) \log(x^2-2) - \int \left(\frac{1}{2} x^2-1\right) \cdot \frac{2x}{x^2-2} \, dx \\
 &= \left(\frac{1}{2} x^2-1\right) \log(x^2-2) - \int x \, dx \\
 &= \left(\frac{1}{2} x^2-1\right) \log(x^2-2) - \frac{1}{2} x^2 + C
 \end{aligned}$$

$$\begin{aligned}
 (8) \int e^x \log(e^x+1) \, dx & \\
 &= \int (e^x+1)' \log(e^x+1) \, dx \\
 &= (e^x+1) \log(e^x+1) - \int (e^x+1) \cdot \frac{e^x}{e^x+1} \, dx \\
 &= (e^x+1) \log(e^x+1) - \int e^x \, dx \\
 &= (e^x+1) \log(e^x+1) - e^x + C
 \end{aligned}$$

$$\begin{aligned}
(9) \int x^2 e^{-x} dx & \\
&= \int x^2 (-e^{-x})' dx \\
&= -x^2 e^{-x} + \int 2x e^{-x} dx \\
&= -x^2 e^{-x} + 2 \int x (-e^{-x}) dx \\
&= -x^2 e^{-x} + 2(-x e^{-x} + \int e^{-x} dx) \\
&= -x^2 e^{-x} - 2x e^{-x} + 2e^{-x} + C \\
&= \underline{- (x^2 + 2x - 2) e^{-x} + C}
\end{aligned}$$

$$\begin{aligned}
(10) \int (\log x)^3 dx & \\
&= \int (x)' (\log x)^3 dx \\
&= x (\log x)^3 - \int x \cdot 3 (\log x)^2 \cdot \frac{1}{x} dx \\
&= x (\log x)^3 - 3 \int (\log x)^2 dx \\
&= x (\log x)^3 - 3 \int (x)' (\log x)^2 dx \\
&= x (\log x)^3 - 3 \left\{ x (\log x)^2 - \int x \cdot 2 \log x \cdot \frac{1}{x} dx \right\} \\
&= x (\log x)^3 - 3x (\log x)^2 \\
&\quad + 6 \int \log x dx \\
&= x (\log x)^3 - 3x (\log x)^2 + 6(x \log x - x) \\
&\quad + C \\
&= x (\log x)^3 - 3x (\log x)^2 + 6x \log x \\
&\quad \underline{- 6x + C}
\end{aligned}$$