(1) $\int (x+1)^3 dx$

$= \frac{1}{4}(x+1)^4 + C$

(2) $\int \sqrt[3]{6x+7}\, dx$

$6x+7=t$ とおくと

$\frac{dt}{dx} = 6$

$dx = \frac{1}{6}dt$

$= \int \sqrt[3]{t} \cdot \frac{1}{6} dt$

$= \frac{1}{6} \cdot \frac{3}{4} \cdot t^{\frac{4}{3}} + C$

$= \frac{1}{8}\sqrt[3]{t^4} + C = \frac{1}{8}\sqrt[3]{(6x+7)^4} + C$

(3) $\int \sin \frac{2\pi}{3} t\, dt$

$\frac{2\pi}{3} t = \theta$ とおくと

$\frac{d\theta}{dt} = \frac{2\pi}{3}$

$dt = \frac{3}{2\pi} d\theta$

$= \int \sin\theta \cdot \frac{3}{2\pi} d\theta$

$= \frac{3}{2\pi}(-\cos\theta) + C$

$= -\frac{3}{2\pi}\cos\theta + C = -\frac{3}{2\pi}\cos\frac{2\pi}{3}t + C$

(4) $\int \frac{dx}{(5x+3)^3}$

$5x+3=t$ とおくと

$\frac{dt}{dx} = 5$ $dx = \frac{1}{5}dt$

$= \int \frac{1}{t^3} \cdot \frac{1}{5} dt$

$= -\frac{1}{10} \cdot \frac{1}{t^2} + C$

$= -\frac{1}{10(5x+3)^2} + C$

(5) $\int 3^{1-x} dx$

$1-x=t$ とおくと

$\frac{dt}{dx} = -1$

$dx = -dt$

$= \int 3^t \cdot (-1) \cdot dt$

$= -\frac{3^t}{\log 3} + C = -\frac{3^{1-x}}{\log 3} + C$

(6) $\int x\sqrt{x+2}\, dx$

$x+2=t$ とおくと

$dx = dt$

$= \int (t-2)\sqrt{t}\, dt$

$= \int (\sqrt{t^3} - 2\sqrt{t})\, dt$

$= \frac{2}{5}t^{\frac{5}{2}} - \frac{4}{3}t^{\frac{3}{2}} + C$

$= \frac{2}{15}t^{\frac{3}{2}}(3t-10) + C$

$= \frac{2}{15}(3x-4)\sqrt{(x+2)^3} + C$

(7) $\int \frac{3x-1}{\sqrt{x+1}} dx$

$x+1=t$ とおくと

$dx = dt$

$3x-1 = 3t-4$

$= \int \frac{3t-4}{\sqrt{t}} dt$

$= \int \left(3\sqrt{t} - \frac{4}{\sqrt{t}}\right) dt$

$= 2t^{\frac{3}{2}} - 8t^{\frac{1}{2}} + C$

$= 2t^{\frac{1}{2}}(t-4) + C$

$= 2(x-3)\sqrt{x+1} + C$

(8) $\int 3x^2(x^3+2)\, dx$

$x^3+2=t$ とおくと

$\frac{dt}{dx} = 3x^2$

$3x^2 dx = dt$

$= \int t\, dt$

$= \frac{1}{2}t^2 + C$

$= \frac{1}{2}(x^3+2)^2 + C$

(9) $\int \sin^2 x \cos x \, dx$

$\sin x = t$ とおくと

$\frac{dt}{dx} = \cos x$

$\cos x \, dx = dt$

$= \int t^2 dt$

$= \frac{1}{3} t^3 + C$

$= \frac{1}{3} \sin^3 x + C$

(10) $\int \frac{dx}{x \log x}$

$\log x = t$

$\frac{dt}{dx} = \frac{1}{x}$

$\frac{dx}{x} = dt$

$= \int \frac{dt}{t}$

$= \log|t| + C$

$= \log|\log x| + C$

(11) $\int \frac{x^2 + 2x}{x^3 + 3x^2 + 1} dx$

$x^3 + 3x^2 + 1 = t$

$\frac{dt}{dx} = 3x^2 + 6x$

$(x^2 + 2x) dx = \frac{1}{3} dt$

$= \int \frac{1}{t} \cdot \frac{1}{3} dt$

$= \frac{1}{3} \log|t| + C$

$= \frac{1}{3} \log|x^3 + 3x^2 + 1| + C$

(12) $\int \frac{\cos x}{1 + \sin x} dx$

$1 + \sin x = t$

$\frac{dt}{dx} = \cos x$

$\cos x \, dx = dt$

$= \int \frac{dt}{t}$

$= \log|t| + C$

$= \log(1 + \sin x) + C$

(13) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

$e^x + e^{-x} = t$

$\frac{dt}{dx} = e^x - e^{-x}$

$(e^x - e^{-x}) dx = dt$

$= \int \frac{1}{t} dt$

$= \log|t| + C = \log(e^x + e^{-x}) + C$

(14) $\int x \cdot \sqrt[3]{1+x} \, dx$

$1 + x = t$

$dx = dt$

$= \int (t-1) \sqrt[3]{t} \, dt$

$= \int (\sqrt[3]{t^4} - \sqrt[3]{t}) dt$

$= \frac{3}{7} t^{\frac{7}{3}} - \frac{3}{4} t^{\frac{4}{3}} + C$

$= \frac{3}{28} t^{\frac{4}{3}} (4t - 7) + C$

$= \frac{3}{28} (4x - 3) \cdot \sqrt[3]{(1+x)^4} + C$

(15) $\int \frac{dx}{\cos^4 x} = \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x} dx$

$= \int \frac{1 + \tan^2 x}{\cos^2 x} dx$

$\tan x = t$

$\frac{dt}{dx} = \frac{1}{\cos^2 x}$

$dt = \frac{dx}{\cos^2 x}$

$= \int (1 + t^2) dt$

$= t + \frac{1}{3} t^3 + C$

$= \tan x + \frac{1}{3} \tan^3 x + C$

(16) $\int (2x+1) e^{x^2+x+5} dx$

$x^2 + x + 5 = t$

$\frac{dt}{dx} = 2x + 1$

$(2x+1) dx = dt$

$\parallel$

$\int e^t dt$

$= e^t + C$

$= e^{x^2+x+5} + C$