

$$1. (1) a_1 = 1, a_2 = 2, a_{n+2} + 3a_{n+1} - 4a_n = 0$$

$$\begin{aligned} x^2 + 3x - 4 &= 0 \\ (x-1)(x+4) &= 0 \\ x &= 1, -4 \end{aligned}$$

$$\Downarrow \\ a_{n+2} - a_{n+1} = -4(a_{n+1} - a_n) \dots \textcircled{1}$$

$$a_{n+2} + 4a_{n+1} = a_{n+1} + 4a_n \dots \textcircled{2}$$

$$\textcircled{1} \text{ の初項は } a_2 - a_1 = 2 - 1 = 1 \text{ より}$$

$$a_{n+1} - a_n = (-4)^{n-1} \dots \textcircled{3}$$

$$\textcircled{2} \text{ の初項は } a_2 + 4a_1 = 2 + 4 = 6 \text{ より}$$

$$a_{n+1} + 4a_n = 6 \dots \textcircled{4}$$

より

$$a_n = \frac{6 - (-4)^{n-1}}{5}$$

$$\textcircled{4} - \textcircled{3} \quad 5a_n = 6 - (-4)^{n-1}$$

$$(2) a_1 = 0, a_2 = 1, a_{n+2} + 5a_{n+1} + 6a_n = 0$$

$$\begin{aligned} x^2 + 5x + 6 &= 0 \\ (x+2)(x+3) &= 0 \\ x &= -2, -3 \end{aligned}$$

$$\Downarrow \\ a_{n+2} + 2a_{n+1} = -3(a_{n+1} + 2a_n) \dots \textcircled{1}$$

$$a_{n+2} + 3a_{n+1} = -2(a_{n+1} + 3a_n) \dots \textcircled{2}$$

$$\textcircled{1} \text{ の初項は } a_2 + 2a_1 = 1 \text{ より}$$

$$a_{n+1} + 2a_n = (-3)^{n-1} \dots \textcircled{3}$$

$$\textcircled{2} \text{ の初項は } a_2 + 3a_1 = 1 \text{ より}$$

$$a_{n+1} + 3a_n = (-2)^{n-1} \dots \textcircled{4}$$

$$\textcircled{4} - \textcircled{3} \quad a_n = \frac{(-2)^{n-1} - (-3)^{n-1}}{1}$$

$$2. a_1 = 2, h_1 = 6, a_{n+1} = 2a_n + h_n \dots \textcircled{1}, h_{n+1} = 3a_n + 4h_n \dots \textcircled{2}$$

$$(1) a_{n+1} + h_{n+1} = 5(a_n + h_n)$$

$$a_1 + h_1 = 8 \text{ より}$$

$$a_n + h_n = 8 \cdot 5^{n-1} \dots \textcircled{3}$$

$$(2) \textcircled{3} + \textcircled{4}$$

$$4a_n = 8 \cdot 5^{n-1}$$

$$a_n = 2 \cdot 5^{n-1}$$

$$3a_{n+1} - h_{n+1}$$

$$= 3(2a_n + h_n) - (3a_n + 4h_n)$$

$$= 3a_n - h_n$$

$$3a_1 - h_1 = 6 - 6 = 0 \text{ より}$$

$$3a_n - h_n = 0 \dots \textcircled{4}$$

$$h_n = 3a_n$$

$$= 6 \cdot 5^{n-1}$$