(1) $a_1 = 1$、$a_{n+1} - a_n = 5$

$d = 5$ より

$a_n = 1 + (n-1)\cdot 5$

$= \underline{5n - 4}$

(2) $a_1 = 5$、$a_{n+1} = -3a_n$

$r = -3$ より

$\underline{a_n = 5\cdot(-3)^{n-1}}$

(3) $a_1 = 2$、$a_{n+1} - a_n = 3n^2 + n$

$n \geqq 2$ のとき

$a_n = 2 + \sum_{k=1}^{n-1}(3k^2 + k)$

$= 2 + \frac{1}{2}n(n-1)(2n-1) + \frac{1}{2}n(n-1)$

$= 2 + \frac{1}{2}(2n^3 - 3n^2 + n) + \frac{1}{2}(n^2 - n)$

$= \underline{n^3 - n^2 + 2}$ ($n=1$ でもok)

(4) $a_1 = 1$、$a_{n+1} = a_n + 4^n$

$n \geqq 2$ のとき

$a_n = 1 + \sum_{k=1}^{n-1} 4^k$

$= 1 + \frac{4(4^{n-1}-1)}{4-1}$

$= 1 + \frac{4^n - 4}{3}$

$= \underline{\frac{4^n - 1}{3}}$ ($n=1$ でもok)

(5) $a_1 = 2$、$a_{n+1} = 3a_n - 2$

$\boxed{\begin{array}{l}\alpha = 3\alpha - 2\\ 2\alpha = 2\\ \alpha = 1\end{array}}$

$\Downarrow$

$a_{n+1} - 1 = 3(a_n - 1)$

$a_1 - 1 = 2 - 1 = 1$

よって、$a_n - 1 = 3^{n-1}$

$\underline{a_n = 3^{n-1} + 1}$

(6) $a_1 = 1$、$a_{n+1} = -2a_n + 9$

$\boxed{\begin{array}{l}\alpha = -2\alpha + 9\\ 3\alpha = 9\\ \alpha = 3\end{array}}$

$\Downarrow$

$a_{n+1} - 3 = -2(a_n - 3)$

$a_1 - 3 = -2$

よって、$a_n - 3 = (-2)^n$

$\underline{a_n = (-2)^n + 3}$

(7) $a_1 = 1$、$a_{n+1} = 3a_n + 4n$

$a_{n+2} - a_{n+1} = (3a_{n+1} + 4n + 4) - (3a_n + 4n)$

$= 3(a_{n+1} - a_n) + 4$

$a_{n+1} - a_n = b_n$ とおくと

$b_{n+1} = 3b_n + 4$

$\boxed{\begin{array}{l}\alpha = 3\alpha + 4\\ 2\alpha = -4\\ \alpha = -2\end{array}}$

$\Downarrow$

$b_{n+1} + 2 = 3(b_n + 2)$

$b_1 + 2 = a_2 - a_1 + 2$

$= 7 - 1 + 2 = 8$

よって、$b_n + 2 = 8\cdot 3^{n-1}$

$a_{n+1} - a_n = b_n = 8\cdot 3^{n-1} - 2$

$n \geqq 2$ のとき $a_n = 1 + \sum_{k=1}^{n-1}(8\cdot 3^{k-1} - 2)$

$= 1 + \frac{8(3^{n-1}-1)}{3-1} - 2(n-1)$

$= 1 + 4(3^{n-1} - 1) - 2n + 2$

$= 4\cdot 3^{n-1} - 2n - 1$ $n=1$ のときok

よって $\underline{a_n = 4\cdot 3^{n-1} - 2n - 1}$

(8) $a_1 = 10$、$a_{n+1} = 2a_n + 2^{n+2}$

$\frac{a_{n+1}}{2^{n+1}} = \frac{a_n}{2^n} + 2$

$\frac{a_n}{2^n} = b_n$ とおくと $b_{n+1} = b_n + 2$、$b_1 = \frac{10}{2} = 5$

よって、$b_n = 5 + (n-1)\cdot 2$

$= 2n + 3$

$\frac{a_n}{2^n} = 2n + 3$

$\underline{a_n = 2^n(2n + 3)}$

(9) $a_1 = 1$、$a_{n+1} = \dfrac{3a_n}{a_n + 3}$

$$\frac{1}{a_{n+1}} = \frac{a_n + 3}{3a_n} = \frac{1}{a_n} + \frac{1}{3}$$

$\dfrac{1}{a_n} = b_n$とおくと $b_{n+1} = b_n + \dfrac{1}{3}$

$$b_1 = \frac{1}{a_1} = 1$$

よって $b_n = 1 + (n-1)\cdot\dfrac{1}{3}$

$$= \frac{1}{3}n + \frac{2}{3}$$

$\dfrac{1}{a_n} = \dfrac{n+2}{3}$ より $\underline{a_n = \dfrac{3}{n+2}}$

(10) $a_1 = \dfrac{1}{2}$、$a_{n+1} = \dfrac{a_n}{2a_n + 3}$

$$\frac{1}{a_{n+1}} = \frac{2a_n + 3}{a_n} = \frac{3}{a_n} + 2$$

$\dfrac{1}{a_n} = b_n$とおくと $b_{n+1} = 3b_n + 2$

$$\Downarrow$$

$\boxed{\begin{array}{l} \alpha = 3\alpha + 2 \\ \alpha = -1 \end{array}}$

$$b_{n+1} + 1 = 3(b_n + 1)$$

$$b_1 + 1 = 2 + 1 = 3$$

よって $b_n + 1 = 3^n$

$$\frac{1}{a_n} = b_n = 3^n - 1$$

$$\underline{a_n = \frac{1}{3^n - 1}}$$

(11) $a_1 = 1$、$na_{n+1} = (n+1)a_n + 1$

$\Downarrow$ $n(n+1)$でわる

$$\frac{a_{n+1}}{n+1} = \frac{a_n}{n} + \frac{1}{n(n+1)}$$

$\dfrac{a_n}{n} = b_n$とおくと $b_{n+1} = b_n + \dfrac{1}{n(n+1)}$

$$b_1 = 1$$

よって $n \geqq 2$のとき

$$b_n = 1 + \sum_{k=1}^{n-1} \frac{1}{k(k+1)}$$

$$= 1 + \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n-1)}$$

$$= 1 + \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n}\right)$$

$$= 1 + 1 - \frac{1}{n} = 2 - \frac{1}{n}$$

$b_n = 2 - \dfrac{1}{n}$ $n = 1$のときもok

$\dfrac{a_n}{n} = 2 - \dfrac{1}{n}$ より

$$\underline{a_n = 2n - 1}$$

(12) $a_1 = 2$、$3na_{n+1} = (n+1)a_n$

$\Downarrow$ $3n(n+1)$でわる

$$\frac{a_{n+1}}{n+1} = \frac{1}{3}\cdot\frac{a_n}{n}$$

$\dfrac{a_n}{n} = b_n$とおくと $b_{n+1} = \dfrac{1}{3}b_n$

$$b_1 = \frac{2}{1} = 2$$

よって $b_n = 2\cdot\left(\dfrac{1}{3}\right)^{n-1}$

$$\frac{a_n}{n} = 2\left(\frac{1}{3}\right)^{n-1}$$

$$\underline{a_n = 2n\left(\frac{1}{3}\right)^{n-1}}$$