1. (1) $y = x^4 - 3x^2 + 2$

$y' = 4x^3 - 6x$

$y'' = 12x^2 - 6$

$\underline{y''' = 24x}$

(2) $y = \dfrac{1}{x+2}$

$y' = -\dfrac{1}{(x+2)^2}$

$y'' = \dfrac{2}{(x+2)^3}$

$\underline{y''' = -\dfrac{6}{(x+2)^4}}$

(3) $y = (2x-1)^4$

$y' = 4(2x-1)^3 \cdot 2 = 8(2x-1)^3$

$y'' = 8 \cdot 3(2x-1)^2 \cdot 2 = 48(2x-1)^2$

$\underline{y''' = 48 \cdot 2(2x-1) \cdot 2 = 192(2x-1)}$

(4) $y = e^{-x}$

$y' = -e^{-x}$, $y'' = e^{-x}$

$y''' = -e^{-x}$ … $\underline{y^{(5)} = -e^{-x}}$

2. (1) $y = \dfrac{1}{x^2-x} = \dfrac{1}{x-1} - \dfrac{1}{x}$

$y' = -\dfrac{1}{(x-1)^2} + \dfrac{1}{x^2}$

$y'' = \dfrac{2}{(x-1)^3} - \dfrac{2}{x^3}$

$y''' = -\dfrac{6}{(x-1)^4} + \dfrac{6}{x^4}$

$= \dfrac{-6\{x^4 - (x-1)^4\}}{x^4(x-1)^4}$

$= \dfrac{-6\{x^2 + (x-1)^2\}\{x^2 - (x-1)^2\}}{x^4(x-1)^4}$

$= \underline{\dfrac{-6(2x^2-2x+1)(2x-1)}{x^4(x-1)^4}}$

(2) $y = \cos^3 x$

$y' = 3\cos^2 x \cdot (-\sin x) = -3\sin x \cos^2 x$

$y'' = -3(\cos^3 x + \sin x \cdot 2\cos x \cdot (-\sin x))$

$= -3\cos^3 x + 6\sin^2 x \cos x$

$= -3\cos^3 x + 6\cos x(1 - \cos^2 x)$

$= -9\cos^3 x + 6\cos x$

$y''' = -9 \cdot 3\cos^2 x \cdot (-\sin x) - 6\sin x$

$= 27\sin x \cos^2 x - 6\sin x$

$= 27\sin x(1 - \sin^2 x) - 6\sin x$

$= \underline{21\sin x - 27\sin^3 x}$

3. (1) $f(x)$の最高次の項を ax^n とする

すると n 次の項は

$-xf'(x) + 3f(x) = -anx^n + 3ax^n + \cdots$

となるので

$-anx^n + 3ax^n$

$= ax^n(-n+3)$

xについての恒等式なので

$\underline{n = 3}$

(2) $f(x) = ax^3 + bx^2 + cx + 1$ とする.

$f'(x) = 3ax^2 + 2bx + c$, $f''(x) = 6ax + 2b$ より

$x(6ax+2b) + (1-x)(3ax^2+2bx+c)$

$+ 3(ax^3+bx^2+cx+1) = 0$

$(9a+b)x^2 + (4b+2c)x + c + 3 = 0$

これより, $a = -\dfrac{1}{6}$, $b = \dfrac{3}{2}$, $c = -3$

よって

$\underline{f(x) = -\dfrac{1}{6}x^3 + \dfrac{3}{2}x^2 - 3x + 1}$