(1) $y = \sin x - \tan x$

$y' = \cos x - \dfrac{1}{\cos^2 x}$

(2) $y = \tan 3x$

$y' = \dfrac{3}{\cos^2 3x}$

(3) $y = \sin x^2$

$y' = \cos x^2 \cdot 2x$

$= 2x \cos x^2$

(4) $y = \cos^3 x$

$y' = 3\cos^2 x \cdot (-\sin x)$

$= -3\cos^2 x \sin x$

(5) $y = \dfrac{1}{\cos x}$

$y' = -\dfrac{-\sin x}{\cos^2 x} = \dfrac{\sin x}{\cos^2 x}$

(6) $y = x \sin 2x$

$y' = \sin 2x + 2x \cos 2x$

(7) $y = \sin 3x \cos 5x$

$y' = \cos 3x \cdot 3 \cdot \cos 5x$

$+ \sin 3x \cdot (-\sin 5x) \cdot 5$

$= 3\cos 3x \cos 5x$

$- 5 \sin 3x \sin 5x$

(8) $y = \sin^5 x \cos 5x$

$y' = 5\sin^4 x \cdot \cos x \cdot \cos 5x$

$+ \sin^5 x \cdot (-\sin 5x) \cdot 5$

$= 5\sin^4 x (\cos x \cos 5x - \sin x \sin 5x)$

$= 5 \sin^4 x \cos 6x$

(9) $y = \sqrt{1+\sin^2 x} = (1+\sin^2 x)^{\frac{1}{2}}$

$y' = \dfrac{1}{2}(1+\sin^2 x)^{-\frac{1}{2}} \cdot 2\sin x \cdot \cos x$

$= \dfrac{\sin 2x}{2\sqrt{1+\sin^2 x}}$

(10) $y = \left(\tan x + \dfrac{1}{\tan x}\right)^2$

$y' = 2\left(\tan x + \dfrac{1}{\tan x}\right) \cdot \left(\dfrac{1}{\cos^2 x} - \dfrac{1}{\sin^2 x}\right)$

$= 2 \cdot \dfrac{1}{\sin x \cos x} \cdot \dfrac{\sin^2 x - \cos^2 x}{\sin^2 x \cdot \cos^2 x}$

$= \dfrac{-16\cos 2x}{(2\sin x \cos x)^3} = \dfrac{-16\cos 2x}{\sin^3 2x}$

(11) $y = \dfrac{1-\sin x}{1+\cos x}$

$y' = \dfrac{-\cos x(1+\cos x) - (1-\sin x)\cdot(-\sin x)}{(1+\cos x)^2}$

$= \dfrac{-\cos x - \cos^2 x + \sin x - \sin^2 x}{(1+\cos x)^2}$

$= \dfrac{\sin x - \cos x - 1}{(1+\cos x)^2}$

(12) $y = \log(x^2+2)$

$y' = \dfrac{1}{x^2+2} \cdot 2x$

$= \dfrac{2x}{x^2+2}$

(13) $y = \log|x^2-4|$

$y' = \dfrac{2x}{x^2-4}$

(14) $y = (\log x)^3$

$y' = 3(\log x)^2 \cdot \dfrac{1}{x} = \dfrac{3(\log x)^2}{x}$

(15) $y = (x\log x - x)^2$

$(x\log x - x)' = \log x + 1 - 1$

$y' = 2(x\log x - x)\cdot \log x$

(16) $y = (x+3)e^{-x}$

$y' = e^{-x} + (x+3)\cdot(-e^{-x})$

$= -(x+2)e^{-x}$

(17) $y = e^x \cos x$

$y' = e^x \cos x + e^x(-\sin x)$

$= e^x(\cos x - \sin x)$

(18) $y = e^{x^2+2x}$

$y' = e^{x^2+2x}\cdot(2x+2)$

$= 2(x+1)e^{x^2+2x}$

(19) $y = \log_a(x^2-1)$

$y' = \dfrac{1}{(x^2-1)\log a}\cdot 2x$

$= \dfrac{2x}{(x^2-1)\log a}$

(20) $y = a^{-3x}$

$y' = a^{-3x}\log a\cdot(-3)$

$= -3a^{-3x}\log a$

(21) $y = e^{-2x}\sin 2x$

$y' = -2e^{-2x}\sin 2x + e^{-2x}\cos 2x\cdot 2$

$= 2e^{-2x}(\cos 2x - \sin 2x)$

(22) $y = \log(\log x)$

$y' = \dfrac{1}{\log x}\cdot\dfrac{1}{x}$

$= \dfrac{1}{x\log x}$

(23) $y = \log_a(\sin x)$

$y' = \dfrac{1}{(\log a)\sin x}\cdot\cos x$

$= \dfrac{\cos x}{(\log a)\sin x}$

(24) $y = \log_a\left(x+\sqrt{x^2-a^2}\right)$

$\left(\sqrt{x^2-a^2}\right)' = \dfrac{2x}{2\sqrt{x^2-a^2}} = \dfrac{x}{\sqrt{x^2-a^2}}$

$y' = \dfrac{1}{\log a\cdot\left(x+\sqrt{x^2-a^2}\right)}\cdot\left(1+\dfrac{x}{\sqrt{x^2-a^2}}\right)$

$= \dfrac{1}{\log a\left(x+\sqrt{x^2-a^2}\right)}\cdot\dfrac{x+\sqrt{x^2-a^2}}{\sqrt{x^2-a^2}} = \dfrac{1}{\log a\cdot\sqrt{x^2-a^2}}$

(25) $y = \log\dfrac{x^2-h}{x^2+h} = \log(x^2-h) - \log(x^2+h)$

$y' = \dfrac{1}{x^2-h}\cdot 2x - \dfrac{1}{x^2+h}\cdot 2x$

$= \dfrac{2x}{x^2-h} - \dfrac{2x}{x^2+h} = \dfrac{2x(x^2+h)-2x(x^2-h)}{(x^2-h)(x^2+h)}$

$= \dfrac{4hx}{(x^2-h)(x^2+h)}$

(26) $y = \dfrac{(1+x)^3(1-2x)}{(1-x)(1+2x)^3}$

$$y' = \frac{\{3(1+x)^2(1-2x) + (1+x)^3\cdot(-2)\}(1-x)(1+2x)^3 - (1+x)^3(1-2x)\{-(1+2x)^3 + (1-x)\cdot 6(1+2x)^2\}}{(1-x)^2(1+2x)^6}$$

$$= \frac{\{3(1-2x) - 2(1+x)\}(1+x)^2(1-x)(1+2x) - (1+x)^3(1-2x)\{-(1+2x) + 6(1-x)\}}{(1-x)^2(1+2x)^4}$$

$$= \frac{(-8x+1)(1+x)^2(1-x)(1+2x) - (1+x)^3(1-2x)(-8x+5)}{(1-x)^2(1+2x)^4}$$

$$= \frac{(1+x)^2\{(-8x+1)(1-x)(2x+1) - (1+x)(-2x+1)(-8x+5)\}}{(1-x)^2(1+2x)^4}$$

$$= \frac{(1+x^2)\{(16x^3-10x^2-7x+1) - (16x^3-2x^2-13x+5)\}}{(1-x)^2(1+2x)^4}$$

$$= \frac{(1+x^2)(-8x^2+6x-4)}{(1-x)^2(1+2x)^4} = \underline{-\frac{2(1+x^2)(4x^2-3x+2)}{(1-x)^2(1+2x)^4}}$$

(27) $y = \dfrac{x}{\sqrt{(a^2+x^2)^3}}$

$$y' = \frac{\sqrt{(a^2+x^2)^3} - x\cdot 3x\sqrt{a^2+x^2}}{(a^2+x^2)^3}$$

$$= \frac{\sqrt{a^2+x^2}\,(a^2+x^2-3x^2)}{(a^2+x^2)^3}$$

$$= \frac{(a^2-2x^2)\sqrt{a^2+x^2}}{(a^2+x^2)^3}$$

$$= \underline{\frac{a^2-2x^2}{(a^2+x^2)\sqrt{a^2+x^2}}}$$

(28) $y = x^{\sin x}$

両辺に log をとる

$\log y = \sin x \cdot \log x$

両辺を x で微分

$\dfrac{y'}{y} = \cos x \log x + \sin x \cdot \dfrac{1}{x}$

$y' = \underline{x^{\sin x}\left(\cos x \log x + \dfrac{\sin x}{x}\right)}$

(29) $y = x^{\frac{1}{x}}$

$\log y = \dfrac{1}{x}\log x$

$\dfrac{y'}{y} = -\dfrac{1}{x^2}\log x + \dfrac{1}{x^2}$

$y' = x^{\frac{1}{x}}\left(\dfrac{1}{x^2} - \dfrac{1}{x^2}\log x\right)$

$= \underline{x^{\frac{1}{x}-2}(1-\log x)}$

(30) $y = (\sin x)^x$

$\log y = x \log(\sin x)$

$\dfrac{y'}{y} = \log(\sin x) + x\cdot\dfrac{1}{\sin x}\cdot\cos x$

$y' = \underline{(\sin x)^x\left\{\log(\sin x) + \dfrac{x}{\tan x}\right\}}$