(1) $\displaystyle \lim_{x\to 0}\frac{\sin 4x}{x} = \lim_{x\to 0}\frac{\sin 4x}{4x}\cdot 4$

$= \underline{4}$

(2) $\displaystyle \lim_{x\to 0}\frac{\sin 2x}{\sin 5x} = \lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\frac{5x}{\sin 5x}\cdot\frac{2x}{5x}$

$= \underline{\dfrac{2}{5}}$

(3) $\displaystyle \lim_{x\to 0}\frac{1-\cos 2x}{x\sin x}$

$\displaystyle = \lim_{x\to 0}\frac{\sin^2 2x}{x\sin x(1+\cos 2x)}$

$\displaystyle = \lim_{x\to 0}\frac{4\sin^2 x\cos^2 x}{x\sin x(1+\cos 2x)}$

$\displaystyle = \lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{4\cos^2 x}{1+\cos 2x} = \underline{2}$

(4) $\displaystyle \lim_{x\to 0}\frac{\sin 3x+\sin x}{\sin 2x}$

$\displaystyle = \lim_{x\to 0}\left(\frac{\sin 3x}{\sin 2x}+\frac{\sin x}{\sin 2x}\right)$

$\displaystyle = \lim_{x\to 0}\left(\frac{2x}{\sin 2x}\cdot\frac{\sin 3x}{3x}\cdot\frac{3x}{2x}+\frac{\sin x}{x}\cdot\frac{2x}{\sin 2x}\cdot\frac{x}{2x}\right)$

$\displaystyle = \frac{3}{2}+\frac{1}{2} = \underline{2}$

(5) $\displaystyle \lim_{x\to \frac{\pi}{2}}\frac{\cos x}{\sin 2x}$

$x-\frac{\pi}{2}=t$ とおくと
$x\to\frac{\pi}{2}$ のとき
$t\to 0$

$\displaystyle = \lim_{t\to 0}\frac{\cos\left(t+\frac{\pi}{2}\right)}{\sin(2t+\pi)}$

$\displaystyle = \lim_{t\to 0}\frac{-\sin t}{-\sin 2t}$

$\displaystyle = \lim_{t\to 0}\frac{\sin t}{2\sin t\cos t} = \underline{\frac{1}{2}}$

(6) $\displaystyle \lim_{x\to 0}\frac{1-\cos x}{\sin x}$

$\displaystyle = \lim_{x\to 0}\frac{\sin^2 x}{\sin x(1+\cos x)}$

$\displaystyle = \lim_{x\to 0}\frac{\sin x}{1+\cos x} = \frac{0}{2} = \underline{0}$

(7) $\displaystyle \lim_{x\to \pi}\frac{\sin(x-\pi)}{x-\pi}$

$x-\pi=t$ とおくと
$x\to\pi$ のとき
$t\to 0$

$\displaystyle = \lim_{t\to 0}\frac{\sin t}{t} = \underline{1}$

(8) $\displaystyle \lim_{x\to \frac{\pi}{2}}\left(x-\frac{\pi}{2}\right)\tan x$

$x-\frac{\pi}{2}=t$ とおくと
$x\to\frac{\pi}{2}$ のとき
$t\to 0$

$\displaystyle = \lim_{t\to 0} t\cdot\tan\left(t+\frac{\pi}{2}\right)$

$\displaystyle = \lim_{t\to 0}\frac{t}{-\tan t} = \underline{-1}$

(9) $\displaystyle \lim_{x\to 1}\frac{\sin \pi x}{x-1}$

$x-1=t$ とおくと
$x\to 1$ のとき
$t\to 0$

$\displaystyle = \lim_{t\to 0}\frac{\sin(\pi t+\pi)}{t}$

$\displaystyle = \lim_{t\to 0}\frac{-\sin \pi t}{\pi t}\cdot\pi = \underline{-\pi}$

(10) $\displaystyle \lim_{x\to 0}\frac{\sin(\sin x)}{\sin x}$

$\sin x=t$ とおくと
$x\to 0$ のとき
$t\to 0$

$\displaystyle = \lim_{x\to 0}\frac{\sin t}{t} = \underline{1}$

(11) $\displaystyle \lim_{x\to 0}\frac{1-\cos 3x}{x^2}$

$\displaystyle = \lim_{x\to 0}\frac{\sin^2 3x}{x^2(1+\cos 3x)}$

$\displaystyle = \lim_{x\to 0}\frac{(\sin 3x)^2}{(3x)^2(1+\cos 3x)}\cdot 9$

$\displaystyle = \lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)^2\cdot\frac{9}{1+\cos 3x} = \underline{\frac{9}{2}}$

(12) $\displaystyle \lim_{x\to 0} x^2\cos\frac{1}{x}$

$-1 \leqq \cos\frac{1}{x} \leqq 1$ で x^2 をかけると

$-x^2 \leqq x^2\cos\frac{1}{x} \leqq x^2$, $\displaystyle \lim_{x\to 0} x^2 = 0$ より

$\displaystyle \lim_{x\to 0} x^2\cos\frac{1}{x} = \underline{0}$