1. (1) $\lim_{t\to 0}(t+1)(2t-3)$

$= 1\cdot(-3) = \underline{-3}$

(2) $\lim_{x\to -2}\dfrac{x+3}{(x+1)(x^2-3)}$

$= \dfrac{1}{-1\cdot 1} = \underline{-1}$

(3) $\lim_{x\to 3}\sqrt{x+1} = \sqrt{4} = \underline{2}$

(4) $\lim_{x\to 0}\dfrac{x^2+3x}{x} = \lim_{x\to 0}(x+3) = \underline{3}$

(5) $\lim_{t\to -2}\dfrac{t^3+8}{t+2}$

$= \lim_{t\to -2}\dfrac{(t+2)(t^2-2t+4)}{t+2}$

$= \lim_{t\to -2}(t^2-2t+4) = \underline{12}$

(6) $\lim_{x\to 0}\dfrac{(x+2)^2-4}{x}$

$= \lim_{x\to 0}\dfrac{(x^2+4x+4)-4}{x}$

$= \lim_{x\to 0}\dfrac{x^2+4x}{x} = \underline{4}$

(7) $\lim_{x\to -1}\dfrac{x^3+x+2}{x^2+x}$

$= \lim_{x\to -1}\dfrac{(x+1)(x^2-x+2)}{x(x+1)}$

$= \lim_{x\to -1}\dfrac{x^2-x+2}{x} = \underline{-4}$

(8) $\lim_{x\to 0}\dfrac{1}{x}\left(\dfrac{6}{x+3}-2\right)$

$= \lim_{x\to 0}\dfrac{1}{x}\cdot\dfrac{-2x}{x+3}$

$= \lim_{x\to 0}\dfrac{-2}{x+3} = \underline{-\dfrac{2}{3}}$

(9) $\lim_{x\to 3}\dfrac{x-\sqrt{2x+3}}{x-3}$

$= \lim_{x\to 3}\dfrac{x^2-(2x+3)}{(x-3)(x+\sqrt{2x+3})}$

$= \lim_{x\to 3}\dfrac{(x+1)(x-3)}{(x-3)(x+\sqrt{2x+3})}$

$= \lim_{x\to 3}\dfrac{x+1}{x+\sqrt{2x+3}} = \dfrac{4}{3+3} = \underline{\dfrac{2}{3}}$

(10) $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x+8}-3}$

$= \lim_{x\to 1}\dfrac{(x-1)(\sqrt{x+8}+3)}{(x+8)-9}$

$= \lim_{x\to 1}\dfrac{(x-1)(\sqrt{x+8}+3)}{x-1}$

$= \lim_{x\to 1}(\sqrt{x+8}+3) = 3+3 = \underline{6}$

(11) $\lim_{x\to -0}\dfrac{x-1}{x^2-3x} = \lim_{x\to -0}\dfrac{x-1}{x(x-3)}$

$x\to -0$ のとき

(分母) $\to +0$、(分子) $\to -1$

となるので

$\lim_{x\to -0}\dfrac{x-1}{x(x-3)} = \underline{-\infty}$

(12) $\lim_{x\to -3-0}\dfrac{x(x+3)}{|2x+6|}$

$= \lim_{x\to -3-0}\dfrac{x(x+3)}{-2(x+3)} = \underline{\dfrac{3}{2}}$

(13) $\lim_{x\to\infty}\dfrac{1}{x+2} = \underline{0}$

(14) $\lim_{x\to -\infty}\left(1-\dfrac{1}{x^3}\right) = \underline{1}$

(15) $\lim_{x\to\infty}(x^2-3x) = \underline{\infty}$

(16) $\lim_{x\to -\infty}(x^2-x^3) = \underline{\infty}$

(17) $\displaystyle\lim_{x\to\infty}\frac{2x+1}{x^2+3x+1}=\underline{0}$

(18) $\displaystyle\lim_{x\to-\infty}\frac{3x^2-5x-2}{x^2-3x+2}=\underline{3}$

(19) $\displaystyle\lim_{x\to\infty}(x+1-\sqrt{x^2+x})$

$\displaystyle=\lim_{x\to\infty}\frac{(x+1)^2-(x^2+x)}{(x+1+\sqrt{x^2+x})}$

$\displaystyle=\lim_{x\to\infty}\frac{x+1}{x+1+\sqrt{x^2+x}}$

$\displaystyle=\lim_{x\to\infty}\frac{1+\frac{1}{x}}{1+\frac{1}{x}+\sqrt{1+\frac{1}{x}}}=\underline{\frac{1}{2}}$

(20) $\displaystyle\lim_{x\to-\infty}(\sqrt{x^2-3x+1}+x)=(*)$

$x=-t$ とおくと $x\to-\infty$ のとき $t\to\infty$

$\displaystyle(*)=\lim_{t\to\infty}(\sqrt{t^2+3t+1}-t)$

$\displaystyle=\lim_{t\to\infty}\frac{(\cancel{t^2}+3t+1)-\cancel{t^2}}{\sqrt{t^2+3t+1}+t}$

$\displaystyle=\lim_{t\to\infty}\frac{3+\frac{1}{t}}{\sqrt{1+\frac{3}{t}+\frac{1}{t^2}}+1}=\underline{\frac{3}{2}}$

(21) $\displaystyle\lim_{x\to-\infty}3^x=\underline{0}$

(22) $\displaystyle\lim_{x\to\infty}\log_5 x=\underline{\infty}$

(23) $\displaystyle\lim_{x\to2}\frac{x-\sqrt{3x-2}}{\sqrt{x+2}-2}=\lim_{x\to2}\frac{(x-\sqrt{3x-2})(x+\sqrt{3x-2})(\sqrt{x+2}+2)}{(\sqrt{x+2}-2)(\sqrt{x+2}+2)(x+\sqrt{3x-2})}$

$\displaystyle=\lim_{x\to2}\frac{(x^2-3x+2)(\sqrt{x+2}+2)}{(x-2)(x+\sqrt{3x-2})}$

$\displaystyle=\lim_{x\to2}\frac{(x-2)(x-1)(\sqrt{x+2}+2)}{(x-2)(x+\sqrt{3x-2})}$

$\displaystyle=\lim_{x\to2}\frac{(x-1)(\sqrt{x+2}+2)}{x+\sqrt{3x-2}}=\frac{2+2}{2+2}=\underline{1}$

(24) $\displaystyle\lim_{x\to0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}=\lim_{x\to0}\frac{(\sqrt[3]{1+x}-\sqrt[3]{1-x})(\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)(1-x)}+\sqrt[3]{(1-x)^2})}{x(\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)(1-x)}+\sqrt[3]{(1-x)^2})}$

$\displaystyle=\lim_{x\to0}\frac{\underline{(1+x)-(1-x)}=2x}{x(\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)(1-x)}+\sqrt[3]{(1-x)^2})}$

$\displaystyle=\lim_{x\to0}\frac{2}{\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)(1-x)}+\sqrt[3]{(1-x)^2}}=\underline{\frac{2}{3}}$

(25) $\displaystyle\lim_{x\to-\infty}\frac{1}{\sqrt{x^2+x+1}-\sqrt{x^2+1}}=\lim_{t\to\infty}\frac{1}{\sqrt{t^2-t+1}-\sqrt{t^2+1}}$

$x=-t$ とおくと
$x\to-\infty$ のとき $t\to\infty$

$\displaystyle=\lim_{t\to\infty}\frac{\sqrt{t^2-t+1}+\sqrt{t^2+1}}{(t^2-t+1)-(t^2+1)}$

$\displaystyle=\lim_{t\to\infty}\frac{\sqrt{t^2-t+1}+\sqrt{t^2+1}}{-t}$

$\displaystyle=\lim_{t\to\infty}\frac{\sqrt{1-\frac{1}{t}+\frac{1}{t^2}}+\sqrt{1+\frac{1}{t^2}}}{-1}=\underline{-2}$

2. (1) $\displaystyle\lim_{x\to1}\frac{a\sqrt{x+1}-b}{x-1}=\sqrt{2}$ … (*)

$x\to1$ のとき（分母）$\to0$ なので（分子）$\to0$ となる

よって $\displaystyle\lim_{x\to1}(a\sqrt{x+1}-b)=0$ より $a\sqrt{2}-b=0$

$b=a\sqrt{2}$ … ①

(*) に ① を代入

$$\lim_{x\to1}\frac{a\sqrt{x+1}-a\sqrt{2}}{x-1}=\lim_{x\to1}\frac{a(\sqrt{x+1}-\sqrt{2})}{x-1}$$

$$=\lim_{x\to1}\frac{a(x+1-2)}{(x-1)(\sqrt{x+1}+\sqrt{2})}$$

$$=\lim_{x\to1}\frac{a}{\sqrt{x+1}+\sqrt{2}}=\frac{a}{\sqrt{2}+\sqrt{2}}=\frac{a}{2\sqrt{2}}$$

よって $\dfrac{a}{2\sqrt{2}}=\sqrt{2}$ より $\underline{a=4,\ b=4\sqrt{2}}$

(2) $\displaystyle\lim_{x\to-1}\frac{\sqrt{x^2+ax}+b}{x^2-1}=\frac{1}{2}$ … (*)

(1) と同様に $\displaystyle\lim_{x\to-1}(\sqrt{x^2+ax}+b)=0$ より $\sqrt{1-a}+b=0$

$b=-\sqrt{1-a}$ … ①

(*) に ① を代入

$$\lim_{x\to-1}\frac{\sqrt{x^2+ax}-\sqrt{1-a}}{x^2-1}=\lim_{x\to-1}\frac{(x^2+ax)-(1-a)}{(x^2-1)(\sqrt{x^2+ax}+\sqrt{1-a})}$$

$$=\lim_{x\to-1}\frac{(x+1)(x+a-1)}{(x+1)(x-1)(\sqrt{x^2+ax}+\sqrt{1-a})}$$

$$=\lim_{x\to-1}\frac{x+a-1}{(x-1)(\sqrt{x^2+ax}+\sqrt{1-a})}$$

$$=\frac{a-2}{-2\cdot2\sqrt{1-a}}=\frac{a-2}{-4\sqrt{1-a}}$$

よって $\dfrac{a-2}{-4\sqrt{1-a}}=\dfrac{1}{2}$

$\cancel{2}(a-2)=-\overset{2}{\cancel{4}}\sqrt{1-a}$

$(a-2)^2=4(1-a)$

$a^2-4a+4=4-4a$

$\underline{a=0,\ b=-1}$