1. (1) $\frac{1}{(3k-2)(3k+1)} = \frac{1}{3}\left(\frac{1}{3k-2} - \frac{1}{3k+1}\right)$ を利用して部分和を求める

$$S_n = \frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \cdots + \frac{1}{(3n-2)(3n+1)}$$

$$= \frac{1}{3}\left(1 - \frac{1}{4}\right) + \frac{1}{3}\left(\frac{1}{4} - \frac{1}{7}\right) + \cdots + \frac{1}{3}\left(\frac{1}{3n-2} - \frac{1}{3n+1}\right)$$

$= \frac{1}{3}\left(1 - \frac{1}{3n+1}\right)$ よって $\lim_{n\to\infty} S_n = \underline{\frac{1}{3}}$

(2) $\frac{1}{\sqrt{3k-2} + \sqrt{3k+1}} = \frac{1}{3}\left(\sqrt{3k+1} - \sqrt{3k-2}\right)$ を利用する

$$S_n = \frac{1}{1+2} + \frac{1}{2+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{10}} + \cdots + \frac{1}{\sqrt{3n-2}+\sqrt{3n+1}}$$

$$= \frac{1}{3}(2-1) + \frac{1}{3}(\sqrt{7}-2) + \frac{1}{3}(\sqrt{10}-\sqrt{7}) + \cdots + \frac{1}{3}\left(\sqrt{3n+1} - \sqrt{3n-2}\right)$$

$= \frac{1}{3}\left(\sqrt{3n+1} - 1\right)$ よって $\lim_{n\to\infty} S_n = \underline{\infty}$

(3) $\frac{2}{1+2+3+\cdots+n} = \frac{2}{\frac{1}{2}n(n+1)} = \frac{4}{n(n+1)} = 4\left(\frac{1}{n} - \frac{1}{n+1}\right)$ より

$$S_n = 2 + \frac{2}{1+2} + \frac{2}{1+2+3} + \cdots + \frac{2}{1+2+3+\cdots+n}$$

$$= 4\left(1 - \frac{1}{2}\right) + 4\left(\frac{1}{2} - \frac{1}{3}\right) + \cdots + 4\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$= 4\left(1 - \frac{1}{n+1}\right)$ よって $\lim_{n\to\infty} S_n = \underline{4}$

2. (1) $\sum_{k=1}^{\infty} 5\left(\frac{\sqrt{3}}{2}\right)^{n-1}$

$$= \frac{5}{1 - \frac{\sqrt{3}}{2}}$$

$$= \frac{10}{2 - \sqrt{3}}$$

$$= \underline{10(2+\sqrt{3})}$$

(2) $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \cdots$

$$= \sum_{k=1}^{\infty}\left(-\frac{3}{4}\right)^{n-1}$$

$$= \frac{1}{1 - \left(-\frac{3}{4}\right)}$$

$$= \frac{1}{1 + \frac{3}{4}} = \frac{1}{\frac{7}{4}} = \underline{\frac{4}{7}}$$

3. (1) 公比 $3x$ なので

$$-1 < 3x < 1$$

よって $-\frac{1}{3} < x < \frac{1}{3}$

和は $\dfrac{1}{1-3x}$

(2) 公比は $3-x$ なので

$$-1 < 3-x < 1$$

よって、$2 < x < 4$、$x=0$

和は $\dfrac{x}{1-(3-x)} = \dfrac{x}{x-2}$

4. (1) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2^n}+\frac{1}{5^n}\right)$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}} + \frac{\frac{1}{5}}{1-\frac{1}{5}}$$

$$= 1 + \frac{1}{4} = \underline{\frac{5}{4}}$$

(2) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n-3}{5^n} = \sum_{n=1}^{\infty}\left\{\left(\frac{2}{5}\right)^n - 3\left(\frac{1}{5}\right)^n\right\}$

$$= \frac{\frac{2}{5}}{1-\frac{2}{5}} - \frac{\frac{3}{5}}{1-\frac{1}{5}}$$

$$= \frac{2}{3} - \frac{3}{4} = \underline{-\frac{1}{12}}$$

(3) $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{3}\right)^n \cos n\pi$

$-1, 1, -1, 1, \cdots$ と繰り返す

$$= \sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^n$$

$$= \frac{-\frac{1}{3}}{1-\left(-\frac{1}{3}\right)} = \underline{-\frac{1}{4}}$$

(4) $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)^n \sin\frac{n\pi}{2}$

n：偶数のとき0

$1, 0, -1, 0, 1, 0, -1, 0, \cdots$ とくり返す

$$= \sum_{n=1}^{\infty}\left(-\frac{1}{3}\right)\cdot\left(-\frac{1}{9}\right)^{n-1}$$

$$= \frac{-\frac{1}{3}}{1-\left(-\frac{1}{9}\right)} = \underline{-\frac{3}{10}}$$

5. 求める無限級数は

$$2 + \left\{2\left(\frac{4}{5}\right) + 2\left(\frac{4}{5}\right)^2 + 2\left(\frac{4}{5}\right)^3 + \cdots\right\} \times 2$$

はじめの高さ　　　上下の分

$$= 2 + \frac{\frac{8}{5}}{1-\frac{4}{5}} \times 2 = 2 + 8 \times 2 = 18$$

<u>18m</u>