1. (1) $\frac{4}{3} > 1$ より正の無限大　　(2) $-1 < -\frac{2}{5} \leqq 1$ より 0に収束

2. (1) $\lim_{n\to\infty} \frac{2^n}{3^n - 4} = \underline{0}$

(2) $\lim_{n\to\infty} \frac{2^{2n}-1}{3^n+5}$

$= \lim_{n\to\infty} \frac{4^n - 1}{3^n + 5} = \underline{\infty}$

(3) $\lim_{n\to\infty} \frac{2^n - (-3)^{n+1}}{(-3)^n + 2^n}$

$= \lim_{n\to\infty} \frac{\left(-\frac{2}{3}\right)^n - (-3)}{1 + \left(-\frac{2}{3}\right)^n}$

$= \underline{3}$

(4) $\lim_{n\to\infty} (7^n - 6^n)$

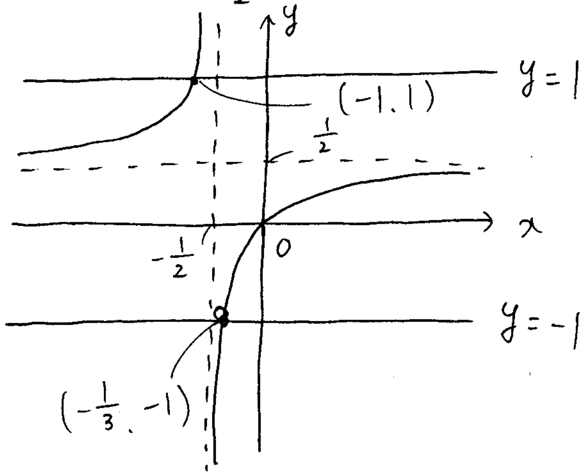
$= \lim_{n\to\infty} 7^n \left(1 - \left(\frac{6}{7}\right)^n\right) = \infty$

3. (1) $-1 < 3x \leqq 1$ より $\underline{-\frac{1}{3} < x \leqq \frac{1}{3}}$

(2) $-1 < \frac{x}{1+2x} \leqq 1$ となる x の範囲を求める.

$y = \frac{x}{1+2x}$ とおくと $y = \frac{x}{1+2x} = -\frac{1}{4} \cdot \frac{1}{x+\frac{1}{2}} + \frac{1}{2}$ となる

$y = -\frac{1}{4} \cdot \frac{1}{x+\frac{1}{2}} + \frac{1}{2}$ のグラフをかくと下の図のようになる.



したがって $-1 < \frac{x}{1+2x} \leqq 1$

となる x の範囲は

$\underline{x \leqq -1,\ -\frac{1}{3} < x}$

4. (1) $a_1 = 1$, $a_{n+1} = \frac{1}{3}a_n + 2$

$\alpha = \frac{1}{3}\alpha + 2$

$\alpha = 3$

$\Downarrow$

$a_{n+1} - 3 = \frac{1}{3}(a_n - 3)$

$a_1 - 3 = 1 - 3 = -2$

よって $\{a_n - 3\}$ は初項 -2,

公比 $\frac{1}{3}$ の等比数列

$a_n - 3 = -2\left(\frac{1}{3}\right)^{n-1}$

$a_n = -2\left(\frac{1}{3}\right)^{n-1} + 3$

よって

$\lim_{n\to\infty} a_n = \underline{3}$

(2) $a_1 = 1$. $a_{n+1} = 2a_n + 1$

$\alpha = 2\alpha + 1$

$\alpha = -1$

$\Downarrow$

$a_{n+1} + 1 = 2(a_n + 1)$

$a_1 + 1 = 1 + 1 = 2$

よって $a_n + 1 = 2^n$

$a_n = 2^n - 1$

$\displaystyle\lim_{n\to\infty} a_n = \underline{\infty}$

(3) $a_1 = \frac{1}{2}$. $a_{n+1} = \dfrac{a_n}{2 + a_n}$

$$\frac{1}{a_{n+1}} = \frac{2 + a_n}{a_n} = \frac{2}{a_n} + 1$$

$\dfrac{1}{a_n} = b_n$ とおくと $b_{n+1} = 2b_n + 1$

$\alpha = 2\alpha + 1$

$\alpha = -1$

$\Downarrow$

$b_{n+1} + 1 = 2(b_n + 1)$

$b_1 + 1 = 2 + 1 = 3$

よって、$b_n + 1 = 3 \cdot 2^{n-1}$

$b_n = 3 \cdot 2^{n-1} - 1$

$$a_n = \frac{1}{3 \cdot 2^{n-1} - 1}$$

$\displaystyle\lim_{n\to\infty} a_n = \underline{0}$

(4) $a_1 = 0$. $a_2 = 1$. $3a_{n+2} = a_{n+1} + 2a_n$

$3x^2 - x - 2 = 0$

$(3x + 2)(x - 1) = 0$

$x = -\frac{2}{3}, 1$

$\Downarrow$

$a_{n+2} - a_{n+1} = -\frac{2}{3}(a_{n+1} - a_n) \cdots ①$

$a_{n+2} + \frac{2}{3}a_{n+1} = a_{n+1} + \frac{2}{3}a_n \cdots ②$

① より $a_{n+1} - a_n = \left(-\frac{2}{3}\right)^{n-1} \cdots ③$

② より $a_{n+1} + \frac{2}{3}a_n = 1 \cdots ④$

④ − ③ $\frac{5}{3}a_n = 1 - \left(-\frac{2}{3}\right)^{n-1}$

$a_n = \frac{3}{5} - \frac{3}{5}\left(-\frac{2}{3}\right)^{n-1}$

$\displaystyle\lim_{n\to\infty} a_n = \underline{\frac{3}{5}}$

(5) $a_1 = 0$. $a_2 = 1$. $a_{n+2} - 7a_{n+1} + 10a_n = 0$

$x^2 - 7x + 10 = 0$

$(x - 2)(x - 5) = 0$

$x = 2, 5$

$\Downarrow$

$a_{n+2} - 5a_{n+1} = 2(a_{n+1} - 5a_n) \cdots ①$

$a_{n+2} - 2a_{n+1} = 5(a_{n+1} - 2a_n) \cdots ②$

① より

$a_{n+1} - 5a_n = 2^{n-1} \cdots ③$

② より

$a_{n+1} - 2a_n = 5^{n-1} \cdots ④$

④ − ③ $3a_n = 5^{n-1} - 2^{n-1}$

$$a_n = \frac{5^{n-1} - 2^{n-1}}{3}$$

$\displaystyle\lim_{n\to\infty} a_n = \underline{\infty}$