1. (1) $\lim_{n\to\infty} \frac{2}{n^3} = \underline{0}$

(2) $\lim_{n\to\infty} \frac{1}{2-n^2} = \underline{0}$

(3) $\lim_{n\to\infty} \left\{1+(-1)^n \underbrace{\boxed{\frac{1}{n^2}}}_{0}\right\}$

$= \underline{1}$

(4) $\lim_{n\to\infty} (1000-\sqrt{n}) = \underline{-\infty}$

(5) $\lim_{n\to\infty} \frac{n}{(-1)^n}$: $\underline{\text{振動}}$

(6) $\lim_{n\to\infty} \sin n\pi = \underline{0}$

(nが自然数のとき、$\sin n\pi = 0$)

2. (1) $\lim_{n\to\infty} \frac{3n^2+4n}{2n^2-3}$

$= \lim_{n\to\infty} \frac{3+\frac{4}{n}}{2-\frac{3}{n^2}}$

$= \underline{\frac{3}{2}}$

(2) $\lim_{n\to\infty} \frac{3n-4}{n^2+1} = \underline{0}$

(3) $\lim_{n\to\infty} (3n^2-2n) = \underline{\infty}$

(4) $\lim_{n\to\infty} \frac{n^2-3n}{5n+4} = \underline{\infty}$

(5) $\lim_{n\to\infty} \frac{\sqrt{2n+1}}{\sqrt{n}}$

$= \lim_{n\to\infty} \sqrt{2+\frac{1}{n}}$

$= \underline{\sqrt{2}}$

(6) $\lim_{n\to\infty} (\sqrt{n^2+1}-n)$

$= \lim_{n\to\infty} \frac{(n^2+1)-n^2}{\sqrt{n^2+1}+n} = \lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}+n} = \underline{0}$

(7) $\lim_{n\to\infty} \frac{2}{\sqrt{n^2+4}-n}$

$= \lim_{n\to\infty} \frac{2(\sqrt{n^2+4}+n)}{(n^2+4)-n^2}$

$= \lim_{n\to\infty} \frac{\sqrt{n^2+4}+n}{2}$

$= \underline{\infty}$

(8) $\lim_{n\to\infty} \frac{1}{n} \cos\frac{n\pi}{4}$

$-1 \leqq \cos\frac{n\pi}{4} \leqq 1$ より

$-\frac{1}{n} \leqq \frac{1}{n}\cos\frac{n\pi}{4} \leqq \frac{1}{n}$

$n\to\infty$ のとき $-\frac{1}{n} \to 0$, $\frac{1}{n} \to 0$

なので、

$\lim_{n\to\infty} \frac{1}{n}\cos\frac{n\pi}{4} = \underline{0}$

(9) $\lim_{n\to\infty} \sqrt{n+1}\left(\sqrt{n+2}-\sqrt{n-1}\right) = \lim_{n\to\infty} \dfrac{\sqrt{n+1}\{(n+2)-(n-1)\}}{\sqrt{n+2}+\sqrt{n-1}}$

$= \lim_{n\to\infty} \dfrac{3\sqrt{n+1}}{\sqrt{n+2}+\sqrt{n-1}}$

$= \lim_{n\to\infty} \dfrac{3\sqrt{1+\frac{1}{n}}}{\sqrt{1+\frac{2}{n}}+\sqrt{1-\frac{1}{n}}} = \dfrac{3}{\sqrt{1}+\sqrt{1}} = \underline{\dfrac{3}{2}}$

(10) $\lim_{n\to\infty} \dfrac{\sqrt{n+5}-\sqrt{n+3}}{\sqrt{n+1}-\sqrt{n}} = \lim_{n\to\infty} \dfrac{\{(n+5)-(n+3)\}\left(\sqrt{n+1}+\sqrt{n}\right)}{\{(n+1)-n\}\left(\sqrt{n+5}+\sqrt{n+3}\right)}$

$= \lim_{n\to\infty} \dfrac{2\left(\sqrt{n+1}+\sqrt{n}\right)}{\sqrt{n+5}+\sqrt{n+3}}$

$= \lim_{n\to\infty} \dfrac{2\left(\sqrt{1+\frac{1}{n}}+\sqrt{1}\right)}{\sqrt{1+\frac{5}{n}}+\sqrt{1+\frac{3}{n}}} = \dfrac{2\left(\sqrt{1}+\sqrt{1}\right)}{\sqrt{1}+\sqrt{1}} = \underline{2}$

(11) $\lim_{n\to\infty} \dfrac{1+2+3+\cdots+n}{n^2} = \lim_{n\to\infty} \dfrac{1}{n^2}\cdot\dfrac{1}{2}n(n+1)$

$= \lim_{n\to\infty} \dfrac{n+1}{2n} = \underline{\dfrac{1}{2}}$

(12) $\lim_{n\to\infty} \dfrac{3+7+11+\cdots+(4n-1)}{3+5+7+\cdots+(2n+1)} = \lim_{n\to\infty} \dfrac{\sum_{k=1}^{n}(4k-1)}{\sum_{k=1}^{n}(2k+1)}$

$= \lim_{n\to\infty} \dfrac{4\cdot\frac{1}{2}n(n+1)-n}{2\cdot\frac{1}{2}n(n+1)+n}$

$= \lim_{n\to\infty} \dfrac{2n^2+n}{n^2+2n} = \underline{2}$