

1. (1) 初項 1, 公差 3 の
等差数列なので, $a_n = 3n - 2$

よって,

$$\sum_{k=1}^n a_k = \sum_{k=1}^n (3k - 2)$$

(2) 初項 1, 公比 3 の等比数列
なので, $a_n = 3^{n-1}$

よって

$$\sum_{k=1}^n a_k = \sum_{k=1}^n 3^{k-1}$$

2. (1) $\sum_{k=1}^n (2k - 2)$

$$= 2 \cdot \frac{1}{2} n(n+1) - 2n$$

$$= \underline{n(n-1)}$$

(2) $\sum_{k=1}^n (3k-1)^2 = \sum_{k=1}^n (9k^2 - 6k + 1)$

$$= 9 \cdot \frac{1}{6} n(n+1)(2n+1)$$

$$- 6 \cdot \frac{1}{2} n(n+1) + n$$

$$= \frac{1}{2} n \{ 3(n+1)(2n+1) - 6(n+1) + 2 \}$$

$$= \frac{1}{2} n \{ 3(2n^2 + 3n + 1) - 6(n+1) + 2 \}$$

$$= \underline{\frac{1}{2} n(6n^2 + 3n - 1)}$$

(3) $\sum_{k=1}^n k(k+1)(2k+1) = \sum_{k=1}^n (2k^3 + 3k^2 + k)$

$$= 2 \cdot \frac{1}{4} n^2(n+1)^2 + 3 \cdot \frac{1}{6} n(n+1)(2n+1) + \frac{1}{2} n(n+1)$$

$$= \frac{1}{2} n^2(n+1)^2 + \frac{1}{2} n(n+1)(2n+1) + \frac{1}{2} n(n+1)$$

$$= \frac{1}{2} n(n+1) \{ n(n+1) + (2n+1) + 1 \}$$

$$= \underline{\frac{1}{2} n(n+1)(n^2 + 3n + 2) = \frac{1}{2} n(n+1)^2(n+2)}$$

3. (1) 第 k 項は

$$2 + 4 + 6 + \dots + 2k$$

$$= \frac{1}{2} k(k+1) \cdot 2 = k(k+1)$$

よって和は

$$\sum_{k=1}^n k(k+1)$$

$$= \sum_{k=1}^n (k^2 + k)$$

$$= \frac{1}{6} n(n+1)(2n+1) + \frac{1}{2} n(n+1)$$

$$= \frac{1}{6} n(n+1) \{ (2n+1) + 3 \}$$

$$= \frac{1}{6} n(n+1) \cdot 2(n+2)$$

$$= \underline{\frac{1}{3} n(n+1)(n+2)}$$

(2) 第 k 項は

$$1 + 3 + 3^2 + \dots + 3^{k-1} = \frac{3^k - 1}{3 - 1}$$

$$= \frac{1}{2} (3^k - 1)$$

よって和は

$$\sum_{k=1}^n \frac{1}{2} (3^k - 1) = \frac{1}{2} \sum_{k=1}^n 3^k - \frac{1}{2} \sum_{k=1}^n 1$$

$$= \frac{3}{2} \sum_{k=1}^n 3^{k-1} - \frac{1}{2} n$$

$$= \frac{3}{2} \cdot \frac{3^n - 1}{3 - 1} - \frac{1}{2} n$$

$$= \underline{\frac{3}{4} (3^n - 1) - \frac{1}{2} n}$$