1. (1) $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^4$

$= \cos\pi + i\sin\pi$

$= \underline{-1}$

(2) $\left\{2\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)\right\}^6$

$= 2^6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$= 64\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= \underline{32 + 32\sqrt{3}i}$

(3) $(1+i)^{12}$

$= \left\{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right\}^{12}$

$= (\sqrt{2})^{12}(\cos 3\pi + i\sin 3\pi)$

$= 2^6 \times (-1) = \underline{-64}$

(4) $\left(\frac{3-\sqrt{3}i}{2}\right)^8 = \left\{\sqrt{3}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)\right\}^8$

$= \left\{\sqrt{3}\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)\right\}^8$

$= (\sqrt{3})^8\left\{\cos\left(-\frac{4}{3}\pi\right) + i\sin\left(-\frac{4}{3}\pi\right)\right\}$

$= 81\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= \underline{\frac{-81 + 81\sqrt{3}i}{2}}$

(5) $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^3 = \left(\frac{1-\sqrt{3}i}{2}\right)^3$

$= \left(\cos\frac{5}{3}\pi + i\sin\frac{5}{3}\pi\right)^3$

$= \cos 5\pi + i\sin 5\pi = \underline{-1}$

(6) $\left(\frac{5-i}{2-3i}\right)^{10} = (1+i)^{10}$

$= \left\{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right\}^{10}$

$= 2^5\left(\cos\frac{5}{2}\pi + i\sin\frac{5}{2}\pi\right)$

$= \underline{32i}$

2. (1) $z^3 = -i$

$z = r(\cos\theta + i\sin\theta)$ とおくと

$z^3 = r^3(\cos 3\theta + i\sin 3\theta)$

また、$-i = \cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi$

よって

$r^3(\cos 3\theta + i\sin 3\theta) = \cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi$

絶対値と偏角を比較して

$r^3 = 1 \qquad 3\theta = \frac{3}{2}\pi + 2k\pi$

$r = 1 \qquad \theta = \frac{1}{2}\pi + \frac{2}{3}k\pi \ (k = 0, 1, 2)$

したがって $\theta = \frac{\pi}{2},\ \frac{7}{6}\pi,\ \frac{11}{6}\pi$

よって $z = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

$z = \cos\frac{7}{6}\pi + i\sin\frac{7}{6}\pi = -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

$z = \cos\frac{11}{6}\pi + i\sin\frac{11}{6}\pi = \frac{\sqrt{3}}{2} - \frac{1}{2}i$

$\underline{z = i,\ -\frac{\sqrt{3}}{2} - \frac{1}{2}i,\ \frac{\sqrt{3}}{2} - \frac{1}{2}i}$

(2) (1)と同様に考えると

$r^4(\cos 4\theta + i\sin 4\theta) = 16(\cos\pi + i\sin\pi)$

$r^4 = 16 \qquad 4\theta = \pi + 2k\pi$

$r = 2 \qquad \theta = \frac{\pi}{4} + \frac{k}{2}\pi \ (k = 0, 1, 2, 3)$

$\theta = \frac{\pi}{4},\ \frac{3}{4}\pi,\ \frac{5}{4}\pi,\ \frac{7}{4}\pi$

よって

$z = \sqrt{2} + \sqrt{2}i,\ -\sqrt{2} + \sqrt{2}i,$

$\underline{-\sqrt{2} - \sqrt{2}i,\ \sqrt{2} - \sqrt{2}i}$