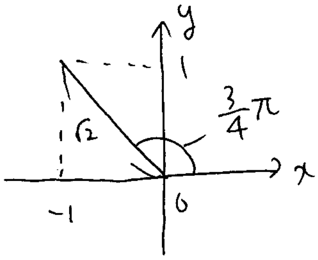
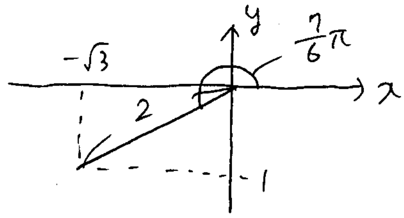
1. (1) $-1+i$

$= \sqrt{2}\left(\cos\frac{3}{4}\pi + i\sin\frac{3}{4}\pi\right)$



(2) $-\sqrt{3}-i$

$= 2\left(\cos\frac{7}{6}\pi + i\sin\frac{7}{6}\pi\right)$



(3) $2+2\sqrt{3}i = 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(4) $-3 = 3\left(\cos\pi + i\sin\pi\right)$

(5) $\sqrt{3} + \dfrac{1-i}{1+i} = \sqrt{3}-i = 2\left(\cos\frac{11}{6}\pi + i\sin\frac{11}{6}\pi\right)$

(6) $\cos\frac{2}{3}\pi - i\sin\frac{2}{3}\pi = \cos\left(-\frac{2}{3}\pi\right) + i\sin\left(-\frac{2}{3}\pi\right)$

$= \cos\frac{4}{3}\pi + i\sin\frac{4}{3}\pi$

2. $\alpha = 1+2\sqrt{2}i$, $\beta = 4-3i$ より $|\alpha| = 3$, $|\beta| = 5$

(1) $|\alpha^4| = |\alpha|^4$
$= 81$

(2) $|\alpha\beta^2| = |\alpha|\cdot|\beta|^2$
$= 75$

(3) $\left|\dfrac{1}{\alpha\beta}\right| = \dfrac{1}{|\alpha|\cdot|\beta|}$
$= \dfrac{1}{15}$

(4) $\left|\dfrac{\beta^2}{\alpha^3}\right| = \dfrac{|\beta|^2}{|\alpha|^3} = \dfrac{25}{27}$

3.

(1) $\dfrac{-1+\sqrt{3}i}{2}z$

$= \left(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right)z$

よって、zを原点を中心として
$\frac{2}{3}\pi$ 回転した点

(2) $(1+i)z$

$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)z$

よって、zを原点を中心として
$\frac{\pi}{4}$ 回転し、絶対値を$\sqrt{2}$倍
にした点

4. 求める点 B は

$$(2-i)\left(\cos\frac{3}{4}\pi+i\sin\frac{3}{4}\pi\right) \text{ or } (2-i)\left\{\cos\left(-\frac{3}{4}\pi\right)+i\sin\left(-\frac{3}{4}\pi\right)\right\}$$

$$(2-i)\left(\cos\frac{3}{4}\pi+i\sin\frac{3}{4}\pi\right) = (2-i)\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)$$

$$= \underline{-\frac{1}{\sqrt{2}}+\frac{3}{\sqrt{2}}i}$$

$$(2-i)\left\{\cos\left(-\frac{3}{4}\pi\right)+i\sin\left(-\frac{3}{4}\pi\right)\right\} = (2-i)\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i\right)$$

$$= \underline{-\frac{3}{\sqrt{2}}-\frac{1}{\sqrt{2}}i}$$

5. $P(x+yi)$ として考える.

$P(x+yi)$ を原点を中心として θ 回転

$$(x+yi)(\cos\theta+i\sin\theta) = \underbrace{x\cos\theta-y\sin\theta}_{\text{実部が}x\text{座標}}+\underbrace{(x\sin\theta+y\cos\theta)}_{\text{虚部が}y\text{座標}}i$$

よって $\underline{Q(x\cos\theta-y\sin\theta,\ x\sin\theta+y\cos\theta)}$

6. $\alpha=3-i$, $\beta=2+3i$,

$$\beta-\alpha=(2+3i)-(3-i)=-1+4i$$

よって $\gamma = (-1+4i)\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)+(3-i)$

$$= (-1+4i)\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)+(3-i)$$

$$= -\frac{\sqrt{3}}{2}-2+2\sqrt{3}i-\frac{1}{2}i+3-i$$

$$= \underline{1-\frac{\sqrt{3}}{2}+\left(2\sqrt{3}-\frac{3}{2}\right)i}$$