

1. 3点 $0, \alpha, \beta$ が一直線上 $\Leftrightarrow \beta = k\alpha$ となる実数 k がある.

$$6 - 4i = k(a + 2i)$$

$$6 - 4i = ka + 2ki$$

$$\text{これより } 6 = ka, -4 = 2k$$

$$\text{よって } k = -2$$

$$\underline{a = -3}$$

2. (1) $|-3 + 4i|$

$$= \sqrt{3^2 + 4^2}$$

$$= \underline{5}$$

(2) $|(1 - 2i)^2| = |-3 - 4i|$

$$= \underline{5}$$

3. (1) $|(7 + 5i) - (3 + 4i)|$

$$= |4 + i|$$

$$= \underline{\sqrt{17}}$$

(2) $|(-1 - 2i) - (-1 + 3i)|$

$$= |-5i|$$

$$= \underline{5}$$

4. (1) $3z + \bar{z} = 2 - 2i \dots \textcircled{1}$ に共役をとると

$$\overline{3z + \bar{z}} = \overline{2 - 2i}$$

$$\underline{3\bar{z} + z = 2 + 2i \dots \textcircled{2}}$$

(2) $\textcircled{1} + \textcircled{2}$ $4z + 4\bar{z} = 4$

$$z + \bar{z} = 1$$

これを $\textcircled{1}$ に代入

$$3z + (1 - z) = 2 - 2i$$

$$2z = 1 - 2i$$

$$\underline{z = \frac{1}{2} - i}$$

5. $|\alpha| = |\beta| = 1$ より $|\alpha|^2 = |\beta|^2 = 1$

$$|\alpha|^2 = 1 \text{ より } \alpha\bar{\alpha} = 1, \quad |\beta|^2 = 1 \text{ より } \beta\bar{\beta} = 1$$

$$\bar{\alpha} = \frac{1}{\alpha}$$

$$\bar{\beta} = \frac{1}{\beta}$$

また $\alpha + \beta + 1 = 0$ に共役をとると $\bar{\alpha} + \bar{\beta} + 1 = 0$

$$\frac{1}{\alpha} + \frac{1}{\beta} + 1 = 0$$

$$\alpha + \beta + \alpha\beta = 0$$

$$\underline{-1}$$

$$\text{よって } \alpha\beta = 1$$

したがって

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (-1)^2 - 2 \cdot 1$$

$$= \underline{-1}$$